Interim
Program of Studies

for

Mathematics 14–24

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(Optional Implementation)

Alberta
LEARNING
Curriculum Standards Branch

MATHEMATICS 14–24

PROGRAM RATIONALE AND PHILOSOPHY

The need for and use of mathematics in our daily lives is changing. This necessitates a change in the emphasis of mathematics education. The emphasis has shifted from the memorization of mathematical formulae and algorithms to the use of mathematics to reason, interpret and explore. The focus is on developing students' mathematical knowledge, skills and attitudes through a problem-solving approach. More than ever before, students must become creative and logical thinkers, data managers and problem solvers. They must develop communication skills and be cooperative and interactive to cope with daily life today and tomorrow.

The use of technology is increasing in importance in our society. This is reflected in our mathematics curriculum. The integration of technology into the mathematics classroom allows students to use computers and calculators to provide quick and accurate computation and manipulation, to enhance conceptual understanding, and to facilitate higher order thinking.

Students have to understand how mathematical concepts permeate daily life, business and industry. They have to be given real-world activities and problems so they can make connections between classroom learning and the world in which they live. Students need to be exposed to varied, interrelated experiences that encourage them to understand and appreciate the role of mathematics in our society.

The Mathematics 14–24 program is derived from *The Common Curriculum Framework for K–12 Mathematics: Western Canadian Protocol for Collaboration in Basic Education*, 1995, and *The Common Curriculum Framework for K–12 Mathematics Grade 10 to Grade 12: Western Canadian Protocol for Collaboration in Basic Education*, 1996. Outcomes have been modified as needed. The Mathematics 14–24 sequence is designed for students whose needs, interests and abilities focus on basic mathematical understanding. The emphasis is on the acquisition of practical life skills and proficiency in using mathematics to solve problems, accommodate change, interpret information and create new knowledge within meaningful contexts.

BELIEFS ABOUT STUDENTS AND MATHEMATICS LEARNING

Students must construct their own meaning of mathematics.

Students are curious, active learners who have individual interests, abilities and needs. They come to classrooms with different knowledge, life experiences and backgrounds that generate a range of attitudes about mathematics and life.

Students learn by attaching meaning to what they do; and they must be able to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of manipulatives can address the diversity of learning styles and developmental stages of students and can enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with appropriate materials, tools and contexts when constructing personal meaning about new mathematical ideas. The learning environment should value and respect each student's way of thinking, so that the learner feels comfortable in taking intellectual risks, asking questions and posing conjectures.

Mathematics is a common human activity, increasing in importance in a rapidly advancing, technological society. Proficiency in using mathematics increases the opportunities available to individuals. Students need to become mathematically literate in order to explore problem-solving situations, accommodate changing conditions and actively create new knowledge in striving for self-fulfillment.

GOALS FOR STUDENTS

Mathematics education must prepare students to use mathematics to solve problems.

The main goals of mathematics education are to prepare students to:

- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- commit themselves to lifelong learning
- become mathematically literate adults, using mathematics to contribute to society.

Positive attitudes toward mathematics are important.

At the completion of a program, students should have developed a positive attitude toward mathematics and have a base of knowledge and skills related to number, patterns and relations, shape and space, and statistics and probability.

It is important for students to develop a positive attitude toward mathematics so that they can become confident in their ability to undertake the problems of a changing world, thereby experiencing the power and usefulness of mathematics. Students also should gain an understanding and appreciation of the contributions of mathematics, as a science and as an art, to civilization and to culture.

Students should:

- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity
- show enjoyment of mathematical experiences.

All students should receive a level of mathematics education appropriate to their needs and abilities.

CONCEPTUAL FRAMEWORK FOR K–12 MATHEMATICS

This framework summarizes the philosophical view toward mathematics and mathematics education.

Students of mathematics, regardless of age or experience, strive to do mathematics in settings that are new to them. The conceptual framework outlined in this section presents a multifaceted view of mathematics and presents the discipline as skills, procedures and concepts woven together.

The framework chart below shows how student outcomes, organized by grade and strand, are designed to be influenced by Mathematical Processes and the Nature of Mathematics. These components are described more fully in this section.

STRAND	Kindergarten to Grade 12	
Number • Number Concepts • Number Operations **Patterns and Relations** • Patterns • Variables and Equations • Relations and Functions **Shape and Space** • Measurement • 3-D Objects and 2-D Shapes • Transformations **Statistics and Probability** • Data Analysis • Chance and Uncertainty	**GENERAL OUTCOMES, SPECIFIC OUTCOMES AND ILLUSTRATIVE EXAMPLES** **to** **Outline Knowledge, Skills and Attitudes about Mathematics**	**NATURE** **OF** **MATHEMATICS** **Change, Constancy, Dimension (size and scale), Number, Pattern, Quantity, Relationships, Shape, Uncertainty**

MATHEMATICAL PROCESSES – COMMUNICATION, CONNECTIONS, ESTIMATION AND MENTAL MATHEMATICS, PROBLEM SOLVING, REASONING, TECHNOLOGY, VISUALIZATION

MATHEMATICAL PROCESSES

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and to encourage lifelong learning in mathematics. Students are expected to:

- *Communication* [C] — communicate mathematically
- *Connections* [CN] — connect mathematical ideas to other concepts in mathematics, to everyday experiences and to other disciplines
- *Estimation and Mental Mathematics* [E] — use estimation and mental mathematics where appropriate
- *Problem Solving* [PS] — relate and apply new mathematical knowledge through problem solving
- *Reasoning* [R] — reason and justify their thinking
- *Technology* [T] — select and use appropriate technologies as tools to solve problems
- *Visualization* [V] — use visualization to assist in processing information, making connections and solving problems.

This program of studies incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning.

Communication

Students must be able to communicate effectively how an answer was obtained.

Students need to communicate mathematical ideas clearly and effectively, orally and in writing.

Communication will help students make connections among different representations of mathematical ideas; namely, "physical, pictorial, graphic, symbolic, verbal and mental representations." (NCTM, p. 26)

NCTM COMMUNICATION STANDARDS

K–4	5–8	9–12
The study of mathematics should include numerous opportunities for communication so that students can:	*The study of mathematics should include opportunities to communicate so that students can:*	*The mathematics curriculum should include the continued development of language and symbolism to communicate mathematical ideas so that all students can:*
• relate physical materials, pictures, and diagrams to mathematical ideas • reflect on and clarify their thinking about mathematical ideas and situations • relate their everyday language to mathematical language and symbols • realize that representing, discussing, reading, writing, and listening to mathematics are a vital part of learning and using mathematics.	• model situations using oral, written, concrete, pictorial, graphical, and algebraic methods • reflect on and clarify their own thinking about mathematical ideas and situations • develop common understandings of mathematical ideas, including the role of definitions • use the skills of reading, listening, and viewing to interpret and evaluate mathematical ideas • discuss mathematical ideas and make conjectures and convincing arguments • appreciate the value of mathematical notation and its role in the development of mathematical ideas.	• reflect upon and clarify their thinking about mathematical ideas and relationships • formulate mathematical definitions and express generalizations discovered through investigations • express mathematical ideas orally and in writing • read written presentations of mathematics with understanding • ask clarifying and extending questions related to mathematics they have read or heard about • appreciate the economy, power, and elegance of mathematical notation and its role in the development of mathematical ideas.
(NCTM, p. 26)	(NCTM, p. 78)	(NCTM, p. 140)

Connections

Through connections students should begin to view mathematics as an integrated whole.

Students need numerous and varied experiences in order to appreciate the usefulness of mathematics and, at the same time, to explore connections within mathematics, from mathematics to other disciplines, and from mathematics to their daily experiences. When mathematical ideas are connected to each other through concrete, pictorial and symbolic representations, students begin to view mathematics as an integrated whole.

This integration "allows students to see how one mathematical idea can help them understand others, and it illustrates the subject's usefulness in solving problems, describing and modeling real-world phenomena, and communicating complex thoughts and information in a concise and precise manner." (NCTM, p. 94)

NCTM CONNECTIONS STANDARDS

K–4	5–8	9–12
The study of mathematics should include opportunities to make connections so that students can:	*The mathematics curriculum should include the investigation of mathematical connections so that students can:*	*The mathematics curriculum should include investigation of the connections and interplay among various mathematical topics and their applications so that all students can:*
• link conceptual and procedural knowledge • relate various representations of concepts or procedures to one another • recognize relationships among different topics in mathematics • use mathematics in other curriculum areas • use mathematics in their daily lives.	• see mathematics as an integrated whole • explore problems and describe results using graphical, numerical, physical, algebraic, and verbal mathematical models or representations • use a mathematical idea to further their understanding of other mathematical ideas • apply mathematical thinking and modeling to solve problems that arise in other disciplines, such as art, music, psychology, science, and business • value the role of mathematics in our culture and society.	• recognize equivalent representations of the same concept • relate procedures in one representation to procedures in an equivalent representation • use and value the connections among mathematical topics • use and value the connections between mathematics and other disciplines.
(NCTM, p. 32)	(NCTM, p. 84)	(NCTM, p. 146)

Estimation and Mental Mathematics

Mental mathematics is the cornerstone for estimation.

Students need to know when and how to estimate. The context of a problem helps to determine when it is necessary or desirable to have an exact answer or an estimate of that answer. Problem contexts include number, patterns and relations, shape and space, and statistics and probability. The use of technology increases the emphasis on estimation skills to enable students to determine the reasonableness of computed answers.

A variety of estimation strategies assists students in arriving at quick approximations for exact answers.

Facility with mental mathematics is an important outcome for students. A focus on mental mathematics forces students to think and improve their efficiency and accuracy in calculating, including pencil and paper calculations. Mental mathematics is the cornerstone for estimation and leads to better understanding of number concepts and number operations. (Hope, pp. 161–173)

Problem Solving

Problem solving is the focus of mathematics at all grade levels.

"*Problem solving—which includes the ways in which problems are represented, the meanings of the language of mathematics, and the ways in which one conjectures and reasons—must be central to schooling so that students can explore, create, accommodate to changed conditions, and actively create new knowledge over the course of their lives.*" (NCTM, p. 4)

Problem solving is the focus of mathematics at all grade levels. The development of each student's ability to solve problems is essential. Students develop a true understanding of mathematical concepts and procedures when they solve problems in meaningful contexts. Problem solving is to be employed throughout all of mathematics and should be embedded throughout all of the strands.

Problem solving provides an opportunity for students to be active in constructing mathematical meaning, to learn problem-solving strategies, to practise a variety of concepts and skills in a meaningful context, and to communicate mathematical ideas. Most problem-solving situations in the elementary years come from the everyday experiences of the student. Students are able to attach mathematical meaning to familiar activities. As they progress through school, the problems become more complex. The problems will arise from an exploration of mathematics itself, as well as from the world around them. Gradually, students become more confident in their ability to use and communicate mathematics, using correct terminology.

As students develop mathematically, they are able to solve more challenging problems on an increasing variety of topics. Students need the opportunity "to solve problems that require them to work cooperatively (and individually), to use technology, to address relevant and interesting mathematical ideas, and to experience the power and usefulness of mathematics." (NCTM, pp. 75–76) By the time students reach the secondary level, many problem-solving strategies should be internalized and problem solving should be a process for constructing and reinforcing mathematical concepts.

Students should be confident and flexible problem solvers, using a wide range of strategies in their work, and accept that some problems have different solutions.

NCTM PROBLEM-SOLVING STANDARDS

K–4	5–8	9–12
The study of mathematics should emphasize problem solving so that students can:	*The mathematics curriculum should include numerous and varied experiences with problem solving as a method of inquiry and application so that students can:*	*The mathematics curriculum should include the refinement and extension of methods of mathematical problem solving so that all students can:*
• use problem-solving approaches to investigate and understand mathematical content • formulate problems from everyday and mathematical situations • develop and apply strategies to solve a wide variety of problems • verify and interpret results with respect to the original problem • acquire confidence in using mathematics meaningfully.	• use problem-solving approaches to investigate and understand mathematical content • formulate problems from situations within and outside mathematics • develop and apply a variety of strategies to solve problems, with emphasis on multistep and nonroutine problems • verify and interpret results with respect to the original problem situation • generalize solutions and strategies to new problem situations • acquire confidence in using mathematics meaningfully.	• use, with increasing confidence, problem-solving approaches to investigate and understand mathematical content • apply integrated mathematical problem-solving strategies to solve problems from within and outside mathematics • recognize and formulate problems from situations within and outside mathematics • apply the process of mathematical modeling to real-world problem situations.
(NCTM, p. 23)	(NCTM, p. 75)	(NCTM, p. 137)

Reasoning

Reasoning helps students to make sense of mathematics and to be logical in their thinking.

Students need to develop confidence in their ability to reason and to justify their thinking within and outside of mathematics. The power of reasoning helps students to make sense of mathematics, to be logical in their thinking, and to convince others.

Inductive reasoning helps students explore and make conjectures from activities that allow generalizations from a pattern of observations.

Deductive reasoning helps students test conjectures and build arguments that serve to validate thinking. Deductive reasoning builds a structured body of knowledge.

NCTM REASONING STANDARDS

K–4	5–8	9–12
The study of mathematics should emphasize reasoning so that students can:	*Reasoning shall permeate the mathematics curriculum so that students can:*	*The mathematics curriculum should include numerous and varied experiences that reinforce and extend logical reasoning skills so that all students can:*
• draw logical conclusions about mathematics • use models, known facts, properties, and relationships to explain their thinking • justify their answers and solution processes • use patterns and relationships to analyze mathematical situations • believe that mathematics makes sense.	• recognize and apply deductive and inductive reasoning • understand and apply reasoning processes, with special attention to spatial reasoning and reasoning with proportions and graphs • make and evaluate mathematical conjectures and arguments • validate their own thinking • appreciate the pervasive use and power of reasoning as a part of mathematics.	• make and test conjectures • formulate counterexamples • follow logical arguments • judge the validity of arguments • construct simple valid arguments.
(NCTM, p. 29)	(NCTM, p. 81)	(NCTM, p. 143)

Technology

Technology will aid students in solving complex problems.

Improvements in technology, and its increased availability in schools, have changed the focus of mathematics education. The time saved by using calculators or computers to perform complex calculations can be used to help students better understand mathematical concepts. Students can then understand the relationships among concepts and use these relationships to solve problems.

Calculators and computers can be used as tools to:

- develop concepts
- explore and demonstrate mathematical relationships and patterns
- organize and display data
- assist with solving problems and thus promote independence
- encourage students to be inquisitive and creative
- decrease the time spent on tedious computations
- reinforce the learning of basic number facts and properties
- develop an understanding of computational algorithms

- create geometric displays
- simulate situations.

In some cases, technology will allow teachers to ask questions requiring a high level of thinking and will allow students to solve complex, multifaceted problems. Technology can foster environments in which the growing curiosity of students can lead to rich mathematical discoveries. In these environments, the control of exploring mathematical ideas can be turned over to students.

Visualization

Images are useful in describing the physical and mathematical environment.

Visualization "involves thinking in *pictures* and *images* and the ability to perceive, transform and re-create different aspects of the visual–spatial world." (Armstrong, p. 10, italics in original) The use of images in the study of mathematics provides students with the opportunity to understand mathematical concepts and to make connections among them.

The physical environment is full of images. The images are of 3-D objects, 2-D shapes, 1-D lines and pictures. In geometry, the study of a 3-D object is assisted by visualizing either the net of 2-D shapes or the skeleton of 1-D lines required to construct the object.

The mathematical environment is full of images. These images are used to communicate mathematical concepts and multiple solutions to problems. At an elementary level, four piles, each containing three coins, can be used to represent $3 + 3 + 3 + 3 = 12$. Rearranging the piles into four rows of 3 can then be used to represent $4 \times 3 = 12$. Connecting the two images links the process of multiplication with that of repeated addition. At a more advanced level, analytic geometry describes figures algebraically and provides a tool for the visualization of algebraic relations. The analysis and interpretation of data, using a visual summary, aids in understanding the data and making predictions from it.

NATURE OF MATHEMATICS

- *Change*
- *Constancy*
- *Dimension*
- *Number*
- *Pattern*
- *Quantity*
- *Relationships*
- *Shape*
- *Uncertainty*

By enriching our view of mathematics and the learning environment, the outcomes of the program of studies can be accomplished.

The brain is constantly looking for and making connections. "Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding. . . . Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching." (Caine, p. 5)

There are additional critical components that must be addressed in a mathematics program beyond those listed as mathematical processes. The components discussed are: Change, Constancy, Dimension (size and scale), Number, Pattern, Quantity, Relationships, Shape and Uncertainty. They are used to describe mathematics in a broad way in order to establish the wide variety of connections that can be made among the various strands used to organize the outcomes central to the program of studies.

Change

Change is a very broad concept. Students must become sensitive to patterns, such as linear, exponential, logarithmic and periodic.

Change can be discussed from Kindergarten to Grade 12 across many aspects of mathematics. The study of change is often discussed in the context of calculus, but is often limited to this context. However, change is a much broader concept than that used in calculus. In order to

make predictions, students need to describe and quantify their observations, attempt to build patterns, and identify those quantities that remain fixed and those quantities that change. For example, look at the pattern 4, 6, 8, 10, 12,.... An elementary school student can describe this as skip counting by 2s, starting from 4. A senior high school student may describe this pattern as an arithmetic sequence, with first term 4, and a common difference of 2. Another student may describe it as a linear function with a discrete domain. All three interpretations are focusing on the changing size of the numbers within the sequence. To be able to understand change, students must become sensitive to patterns, such as linear, exponential, logarithmic and periodic. (Steen, p. 184)

Constancy

Constancy is described by the terms stability, conservation, equilibrium, steady state and symmetry.

Students are expected to communicate ideas visually, using diagrams and oral and written words, when describing constancy or invariance. Different aspects of constancy "are described by the terms stability, conservation, equilibrium, steady state, and symmetry." (AAAS–Benchmarks, p. 270) The most important properties in mathematics and science relate to those properties that do not change when outside conditions change. Elementary school students deal with constancy in situations where different methods are used to solve a single multiplication problem, such as finding the area of a 3-tile by 4-tile tabletop. Secondary students need to deal with constancy when they solve the more complicated multiplication problems that appear in determining the number of elements present in the sample spaces of probability problems. Many of these situations will involve permutations and combinations.

In geometry, a circle can be transformed into an ellipse by a simple stretch, and into a square by a more complex series of transformations; but there is no way that the circle can be transformed into a parabola. The closed figures, such as circles and squares, remain closed and cannot be transformed into open figures, such as parabolas. Triangles can be distorted in many ways, but all will have an angle sum of 180°. The straight line is characterized as having all its parts with the same slope. In solving many of the most important problems in mathematics, students need to concentrate on the properties that remain constant. This idea enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations, or the angle sums of polygons.

Dimension (size and scale)

The concept of dimension needs to be developed within an environment of physical objects.

The concept of dimension, most usually associated with 3-D objects, 2-D shapes or 1-D lines, needs to be developed within an environment of physical objects for all grades from Kindergarten to Grade 12. The prediction of the change in dimension of objects can be done using numbers attached to appropriate units. For example, with no knowledge of a formula, students in upper elementary grades can predict that doubling the side of a square generates four times the area. Junior and senior high school students need to be able to use algebraic structures to formalize this relationship.

Physical objects can all be described using measurement concepts. The development of perimeter, area and volume concepts relies on pattern recognition, not on memorization of formulas. Descriptions of geometric patterns (number of vertices, sides and edges of various 3-D objects, 2-D shapes and 1-D lines); and the angle sum of various 2-D shapes is also encouraged. This type of data should be placed in charts and/or graphs to help students visualize their findings and predict patterns.

Number

The use of number must include number sense.

Number, number systems and the operations on numbers are vital to all mathematics learning. The use of number must go beyond procedure and accuracy to include what is called number sense. Number sense includes:

- an intuitive feeling about numbers and their multiple relationships
- construction of the meaning of number through a variety of experiences, and development of an appreciation of the need for numbers beyond whole numbers (NCTM, p. 38)
- an appreciation and ability to make quick order of magnitude approximations (Steen, p. 79) with emphasis on establishing quick and accurate estimations for computation and measurement
- the ability to detect arithmetic errors
- knowledge of place value and the effects of arithmetic operations.

Many numerical calculations are performed with calculators and computers, and students must be able to determine if the desired calculations have been done correctly. Students must plan for the efficient use of technological tools.

Number patterns should be recognized and used to count, to make predictions, to describe shapes and to compare.

Pattern

Mathematics is an exploratory science that seeks to understand every kind of pattern.

"What humans do with the language of mathematics is to describe patterns. Mathematics is an exploratory science that seeks to understand every kind of pattern." (Steen, p. 8) Patterns exist in number, geometry, algebra and data. By helping students recognize, extend, create and use patterns as a routine aspect of their lives, mathematics will become a useful tool to assist them in their systematic and intellectual understanding of their environment.

Quantity

Quantitatively literate people use numbers to describe phenomena in all aspects of mathematics.

"Quantitatively literate young need a flexible ability to identify critical relations in novel situations, to express these relations in effective symbolic form, to use computing tools to process information, and to interpret the results of those calculations." (Steen, p. 65)

Students have a strong desire to measure, code and order things. To this end, some of the outcomes are about single numbers, numbers attached to units of measure, and ordered sets of numbers. Other outcomes are about the interpretation of numbers and of number systems. The use of single numbers and of ordered pairs to describe phenomena in all aspects of mathematics, the natural sciences and the social sciences is very important.

With the growing use of technology to process numerical information, it is becoming essential for students to have a wide range of estimation skills so that they can evaluate whether or not the numerical output provided by a calculator or a computer is a reasonable solution to a given problem.

Relationships

The study of mathematics is the development of relationships between and among things.

The study of mathematics is the development of relationships between and among things. Part of mathematics should help students develop a sense of discovery that mathematicians over the years have felt and should prepare the way for students to make their own discoveries. Students should look for relationships among physical things, as well as the data used to describe those things. Descriptions of the attributes of objects are used to analyze symmetry and congruence and to classify things, using increasingly sophisticated language. Relationships will be described visually, symbolically, orally and in written form.

Shape

Shape in mathematics includes geometric representations of algebraic relations, the geometry of maps and the creation of networks of figures.

Shape in mathematics is central to geometry but also includes geometric representations of algebraic relations, the geometry of maps and the creation of networks of plane figures that can be used to construct 3-D objects. It is very important for students to look for and use similarities, congruences, patterns, transformations, dilatations and tessellations in the solution of a range of problems.

The use of language to describe shapes is an important aspect of mathematics. This description allows for the classification of objects according to various attributes, the naming of objects, and the analysis of objects. The study of shape can be used to build a deductive system, which can assist in further, more detailed analysis. Shape is used in the development of visual models in other disciplines, such as the use of molecular models in chemistry and biology.

The use of technology to analyze and depict shape will increase in importance for students of mathematics as more and better software and hardware become available in classrooms.

Uncertainty

Uncertainty involves data, chance, measurements and errors.

Uncertainty involves data, chance, measurements and errors. Problems dealing with data, together with numbers in context found in the mass media, can be solved within the school mathematics program so long as the data provided and the problems posed have some meaning and relevance to students.

Chance deals with the predictable and the unpredictable outcomes of events. Students from an early age are expected to deal with the concept of chance. As they mature, the language they use to describe chance becomes more sophisticated and involves the vocabulary of probability theory.

When dealing with random events and complex experiments, students can generate large quantities of data requiring analysis. The use of various technologies enables the student to summarize data easily and to create a visualization of the data to help identify patterns in the information. In some instances the functions describing patterns are linear, periodic, logarithmic or exponential, and senior high school students are expected to use the appropriate algebraic structures to model the information contained within the pattern.

The quality of the output information is directly related to the quality of the input data. The study of uncertainty allows students to assess the reliability of input data, and to learn the processes whereby input data is converted to output information.

STRANDS

- *Number*
- *Patterns and Relations*
- *Shape and Space*
- *Statistics and Probability*

The student outcomes are organized within four strands. The strands are the formal aspects of the discipline of mathematics that form the foundation of the program of studies and act as connections across the grades. Four strands have been identified for the program of studies to reinforce the interrelationship of mathematical concepts and skills. These strands are split into substrands. However, any such grouping into strands and substrands is for organizational purposes only, and does not reflect the connections among the strands and the underlying themes running throughout all of mathematics.

Number

Number Concepts

Students will:

- use numbers to describe quantities
- represent numbers in multiple ways.

Number Operations

Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

Patterns and Relations

Patterns

Students will:

- use patterns to describe the world and to solve problems.

Variables and Equations

Students will:

- represent algebraic expressions in multiple ways.

Relations and Functions

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

Shape and Space

Measurement

Students will:

- describe and compare everyday phenomena, using either direct or indirect measurement.

3-D Objects and 2-D Shapes

Students will:

- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

Transformations

Students will:

- perform, analyze and create transformations.

Statistics and Probability

Data Analysis

Students will:

- collect, display and analyze data to make predictions about a population.

Chance and Uncertainty

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

STUDENT EXPECTATIONS

The content of the program of studies is stated in terms of outcomes. These outcomes are measurable and identify what students are required to know and be able to do.

The outcomes are developed and based on the expectation that they are appropriate to a large majority of the students. There may be some time delays between where students first encounter the learning and where they are expected to demonstrate knowledge of, or mastery in, that learning.

Student expectations are described in terms of:

- *general outcomes*
- *specific outcomes*
- *illustrative examples.*

General Outcomes

General outcomes are general statements that identify what students are expected to know and be able to do upon completion of a course.

Specific Outcomes

Specific outcomes are statements identifying the component knowledge, skills and attitudes of a general outcome.

Illustrative Examples

Illustrative examples are sample tasks that demonstrate and elaborate on the general and specific outcomes. They are important in conveying the richness, breadth and depth intended in the outcomes. The illustrative examples are for discretionary use.

SUMMARY

The components of the conceptual framework for Kindergarten to Grade 12 mathematics, as described, dictate what should be happening in mathematics education. The components are not meant to stand alone, but are to be interrelated to enhance one another. Activities that take place in the classroom should stem from a problem-solving approach built upon the seven mathematical processes. These processes lead students to an understanding of the nature of mathematics through specific knowledge, skills and attitudes related to each of the strands.

INSTRUCTIONAL FOCUS

SUGGESTED TIME ALLOTMENTS

The program of studies is arranged into four strands, each of significance. Therefore, considerable time should be spent on the concepts and processes identified in each strand.

Several additional considerations are important.

- Integration of the mathematical processes, within each strand, is encouraged and expected.
- By decreasing emphasis on rote calculation, drill and practice, and the size of numbers used in paper and pencil calculations, more time is available for concept development.
- Problem solving, reasoning and connections are vital to increasing mathematical power and must be integrated throughout the program. A minimum of half the available time within all strands needs to be dedicated to activities related to these processes.
- There is to be a balance between estimation and mental mathematics, paper and pencil exercises and the appropriate use of technology, including calculators and computers. Concepts should be introduced, using manipulatives, and gradually developed from the concrete to the pictorial to the symbolic.
- There is an assumption made that all students have regular access to appropriate technology. For Mathematics 14–24, calculators and standard spreadsheet programs are appropriate.

CODING OF OUTCOMES FOR MATHEMATICS 14–24

The specific outcomes for Mathematics 14 and Mathematics 24 have been numbered sequentially within each strand. Each specific outcome for Mathematics 24 has been cross-referenced to the Common, Applied or Pure outcomes of *The Common Curriculum Framework for K–12 Mathematics Grade 10 to Grade 12: Western Canadian Protocol for Collaboration in Basic Education*, 1996. For example, (A2–1) refers to Cluster Applied A2, Specific Outcome Number 1.

Strand: Number (Number Concepts)
Students will:
- use numbers to describe quantities
- represent numbers in multiple ways.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]

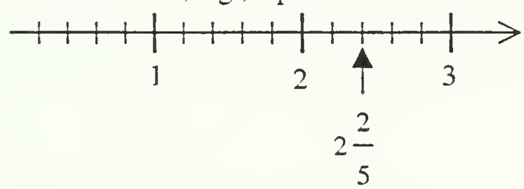
General Outcome

Develop and demonstrate a number sense for rational numbers, decimals, common fractions, integers and whole numbers.

Specific Outcomes

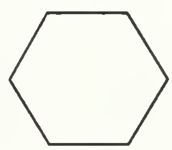
1. Demonstrate and explain the meaning of fractions, concretely, pictorially and symbolically. [C, CN, R, V]

1.1 a) Have students work with fractions that represent:
- part of a group; e.g., $\frac{1}{3}$ of the group were boys
- part of a whole; e.g., $\frac{1}{2}$ of the pizza has pineapple on it
- a measurement; e.g., a point on a number line



b) Have students work with common fractions; e.g., halves, thirds, fourths, fifths, sixths, eighths, tenths, hundredths.
c) Have students use manipulatives to demonstrate that fractions may be less than one or greater than one.

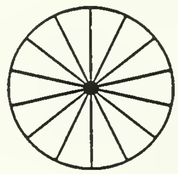
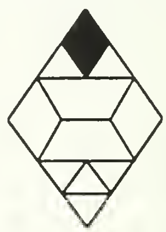
1.2 Show that one half is equivalent to three-sixths, using the following figure.



Add and trace more pattern blocks to make a new figure that shows:
a) one fifth is equivalent to two-tenths
b) six-eighths is equivalent to three-fourths.

Strand: Number (Number Concepts)
Students will:
- use numbers to describe quantities
- represent numbers in multiple ways.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
	1.3 Use fraction circles to show one half. What equivalent names match your choices? Give three other names equivalent to one half. Identify fraction circles that show two-thirds. Name a fraction that is greater than one half but less than two-thirds. How could you use fraction circles to show your reasoning?  1.4 Let the base-10 flat represent one whole square. Use centimetre cubes to build a one-layer shape that is less than one whole square. Record your shape on cm grid paper. Write a fraction and a decimal to tell how much of the square is covered by your shape and how much is not covered by your shape. 1.5 With the help of pattern blocks, and using the value of the yellow hexagon as one whole, make a pattern with a value of $2\frac{1}{2}$. 1.6 The following diagram is made from pattern block pieces. If the shaded area has a value of $\frac{1}{3}$, what is the value of the whole design? Make a design with a value of eleven-thirds. Rearrange the blocks to show eleven-thirds as a mixed number. 

Strand: Number (Number Concepts)
Students will:
- use numbers to describe quantities
- represent numbers in multiple ways.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
2 Demonstrate and explain the meaning of ratio concretely, pictorially and symbolically. [C, CN, R, V]	2.1 Dennis linked 10 cubes together in this order: G G G G Y G G G G Y What ratio describes: 2:8 4:1 8:10 Suppose Dennis continued the pattern to cover a 10 by 10 grid. How would the ratios change? How would you use per cent to describe: a) the green area? b) the area not green? c) the yellow area? Would it be true to write the following about the yellow area? $0.20 = \frac{2}{10} = \frac{1}{5}$ Explain. Sketch and label a 5-cube sequence. Describe the colours in different mathematical ways. 2.2 A fruit punch is made by mixing 3 L of pop with 1 L of orange juice. Model the situation with two different coloured tiles. Find the amount of pop required for 2 L, 3 L and 4 L of orange juice. Write each case as a ratio.
3. Represent and apply fractions as per cents, and per cents in fraction or decimal form. [CN, E, PS, R, T]	3.1 Have students bring in examples of ratios, per cents, decimals and fractions from newspapers or magazines, or by searching the Internet. Discuss with the class why the chosen form was used in each case. Does one form tend to be used more often in particular types of media? Adapted with permission of the British Columbia Ministry of Education.

Strand: Number (Number Concepts)
Students will:
- use numbers to describe quantities
- represent numbers in multiple ways.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]

3.2 Give students lists of fractions and decimals to arrange in numerical order and place on a number line. Have students develop lists of rules based on their work, which they can use to place numbers in order.
a) Can students correctly order and place numbers?
b) Do they make conversions correctly?
c) Do the rules they identify accurately reflect the process?

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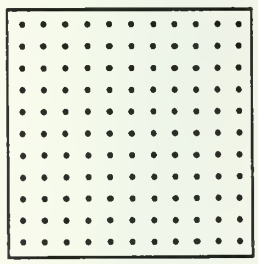
3.3 Have students use a chart to represent numbers taken from real-life data in as many ways as possible; e.g., ratio, fraction, per cent, decimal.

3.4 Have students practise calculating common percentages mentally; e.g., 7%, 15%, 25%, 50%.
a) What is the GST on a $90 pair of running shoes?
b) If 25% of the 600 people at a basketball game are with the visiting team, how many people is this?

3.5 Simulation: The stock market. Choose ten stocks that students are familiar with from the Toronto Stock Exchange. Give students a fictional amount of money to invest and have them track the movement of the stocks each day. Then, by converting to decimal fractions and per cents, have them calculate how much money they have made or lost.

Adapted with permission of the British Columbia Ministry of Education.

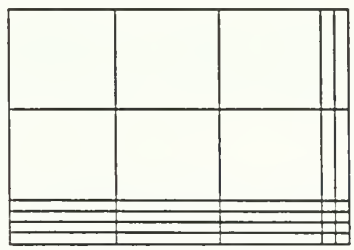
3.6 Let the largest possible square on an 11 by 11 pin geoboard have a value of 1. Construct a different (noncongruent) shape for each part named below:
a) 0.25 of the square
b) $\frac{1}{4}$ of the square
c) 25% of the square.



Record, colour and label each shape on geodot paper. How is each coloured part the same?
Shannon outlined a new shape. She says the ratio of the coloured part to the whole square is 3:5. Record and colour one possible shape Shannon might have used. Record other ways to name this shape as part of 1.

Strand: Number (Number Operations)
Students will:
- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Apply arithmetic operations on common fractions, decimals and integers, and illustrate their use in solving problems. **Specific Outcomes**	
4. Use patterns, manipulatives and diagrams to demonstrate the concepts of multiplication and division by a decimal. [C, CN, PS, R, V]	4.1 Revalue the base-10 blocks. For example, let the "flat" represent one. Then the "long" represents one tenth, and the "unit" represents one hundredth. The diagram below shows the multiplication 3.2×2.4.  Use base-10 blocks and this diagram to find the product, and explain your procedure. 4.2 Jacques bought 13.2 m of fabric to make table cloths. Each cloth takes 2.4 m of fabric. How many table cloths can he make? Use base-10 blocks to find the answer. Explain how your answer is related to the answer you get on your calculator.
5. Use estimation strategies to justify or assess the reasonableness of calculations. [C, E, PS, R]	5.1 Liam bought 3 equally priced CDs and a tape priced at \$11.95. He paid \$71.20 in total. Before you calculate the exact cost of each CD, explain why it must be less than \$20.00. What was the cost of each CD? Estimate the GST on the total.
6. Add, subtract, multiply and divide decimals (for more than 2-digit divisors or multipliers, the use of technology is expected). [E, PS, T]	6.1 Apples cost \$1.39 per kilogram. If you have \$6.00, can you purchase a 3.75 kg bag of these apples? Explain how you could make an estimate to be sure, before you purchase the apples. Find the exact cost of the apples.

Strand: Number (Number Operations)
Students will:
- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
7. Add, subtract, multiply and divide integers concretely, pictorially and symbolically. [PS, V]	7.1 Show how you can use two different coloured cubes to represent the following numbers, and combine them. $(^{+}10) + (^{-}6) =$ $(^{-}4) + (^{-}7) =$ $(^{-}8) + (^{+}5) =$ 7.2 Calvin put equal numbers of white and black cubes in a container to make a neutral charge. He then put in 6 white cubes to make a charge of -6. From this container he removed four black cubes and wrote $(^{-}6) - (^{+}4) = (^{-}10)$. Show why this is true and use a similar method to find: $(^{+}5) - (^{-}2)$ $(^{-}3) - (^{-}5)$ $(^{+}7) - (^{+}6)$. 7.3 Hoang explained $(^{+}5) \times (^{-}2)$ as <u>putting</u> 5 groups of 2 white chips each <u>into</u> a container for a product of $^{-}10$. He explained $(^{-}6) \times (^{+}4)$ as <u>taking</u> 6 groups of 4 black chips <u>out</u> of a neutral container for a product of $^{-}24$. Using Hoang's reasoning, demonstrate and explain $(^{-}3) \times (^{-}5)$ and $(^{+}7) \times (^{+}6)$. 7.4 The temperature fell 2°C per hour for a total temperature change of -10°C. How many hours did this take? 7.5 Extend the following pattern, and describe the rule that is used. $-3, 6, -12, \ldots$ 7.6 Provide an answer and have students suggest as many questions as they can that would result in that answer; e.g.: A = 6, Q = 2 + 4, 3 x 2, 20 – 14, 24 ÷ 4 . . . Adapted with permission of the British Columbia Ministry of Education.
8. Illustrate and explain the order of operations, using paper and pencil or a calculator. [PS, T, V]	8.1 Determine the missing operation signs so that the following statement is true. $(7.4 \square 2.1) \square 14 = 1.11$ 8.2 Place parentheses in the following equation to make it true. $4 + 5 \times 3 - 8 = 19$

Strand: Number (Number Operations)
Students will:
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[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
9. Add, subtract, multiply and divide fractions concretely, pictorially and symbolically. [E, PS, V]	9.1 Eric ordered several large pizzas for a party. $1\frac{1}{2}$ pepperoni pizzas and $\frac{2}{3}$ of a pineapple pizza were not eaten. Was there the equivalent of more than two large pizzas left over? Explain how you can estimate the answer. Solve using a pencil and paper method, and use paper circles to explain your method and your answer. 9.2 Mr. Blair's gas tank was $\frac{7}{8}$ full when he left home. He knows he will use $\frac{3}{4}$ of a tank of gas on his errands. Will he have enough fuel to do his errands. Use fraction strips to explain your method and your answer. 9.3 Lisa had $\frac{3}{4}$ of a large candy bar. She gave $\frac{1}{3}$ of what she had to Shannon. Explain how you know that Shannon got less than $\frac{1}{3}$ of a whole bar by: a) using a pencil and paper method b) folding a piece of paper that represents a whole candy bar. 9.4 Miko has $2\frac{1}{2}$ m of blue cloth. How many pieces $\frac{1}{4}$ m long can she cut from her piece? Estimate the answer and explain the solution by: a) using a pencil and paper method b) using manipulatives. 9.5 In the community hall, $\frac{1}{4}$ of the people present are men, $\frac{1}{3}$ are women and the rest are children. There are 840 people in the hall. How many children are there?

Strand: Number (Number Operations)
Students will:
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[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[E] Estimation and Mental Mathematics	[T] Technology
	[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
	9.6 Provide students with a recipe that contains fractions; e.g., $\frac{1}{3}$ cup, and ask students how they would double or triple the recipe. Have them determine when they would use mathematics to calculate new amounts of ingredients and when they would simply duplicate the original amounts. How would they halve the recipe? Have students use measuring cups and water to represent the situation and verify solutions. Adapted with permission of the British Columbia Ministry of Education. 9.7 Invite students to play games and create their own games that incorporate operations with rationals; e.g., mathematics crossword, mathematics bingo, who am I? Adapted with permission of the British Columbia Ministry of Education.
10. Estimate, compute and verify the sum, difference, product and quotient of rational numbers, using only decimal representations of negative rationals. [E, PS, T]	10.1 The temperature in the morning was 7°C. By noon the temperature went up 9°C. By late afternoon it fell 5°C, and by midnight it dropped another 9°C. What is the final temperature?

Strand: Number (Number Operations)
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[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Apply the concepts of rate, ratio, percentage and proportion to solve problems in meaningful contexts. **Specific Outcomes** 11. Estimate and calculate percentages. [E, PS, T]	11.1 Les Krantz, in his book called *What the Odds Are* found that men will suffer severe hair loss—baldness—by the time they reach the age ranges listed below: 20–29: 1 in 5 30–39: 3 in 10 40–49: 2 in 5 50–59: 1 in 2 60–69: 2 in 3 70–79: 3 in 4 How many men in every hundred would you expect to have lost most of their hair by the time they reach these age ranges? a) 20 to 29 b) 40 to 49 c) 60 to 69 Predict the percentage of men between the ages of 80 to 89 who will be bald. Explain your reasoning for your prediction. 11.2 Donna works at a local hardware store, making $25 000 per year. In March, her boss requests that she take a 5% salary reduction, as business is slow and she needs to stabilize the store's financial position. Donna agrees, and by June the situation has improved; her boss awards her a 5% salary increase. Has Donna's salary returned to the $25 000 level? Explain your answer. 11.3 Almost 14% of Canada's land surface is covered by wetlands. If Canada's land surface is 1020 million hectares, how many hectares are covered by wetlands?

Strand: Number (Number Operations)
Students will:
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[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
12. Derive and apply unit rates. [PS, R]	12.1 A carpenter bought 2.5 kg of nails. The total cost was \$3.75. What was the cost of 1 kg? 12.2 Toothpaste is advertised as 75¢ for a 50 mL tube. A 75 mL tube is priced at \$1.09. Which is the better buy? Why? 12.3 Compare values, using flyers from two grocery outlets. 12.4 Have students compare the cost of 1 kg of bulk peanuts with the cost of 1 kg of peanuts bought in smaller packages. 12.5 Travelling from Saskatoon to Regina, a distance of 276 km, took 3 hours. What was the rate of speed in km/hr? 12.6 Mark drove from Seattle to Portland, a distance of 124 miles, in 2 hours. What was his rate of speed in miles per hour?
13. Express rates and ratios in equivalent forms to solve problems. [PS, R, T]	13.1 Gas usage is expressed as the rate of the number of litres of gas used per 100 km. On a 225 km trip, Nadia used 20.5 L of gas. Express her usage in terms of the above rate. Why do you think this type of rate is used? 13.2 While visiting the California coast, Theresa drove 270 miles. If she used $11\frac{1}{2}$ gallons of gas, what was her rate of gas use? Express your answer in miles per gallon. 13.3 In Canada, there are 1 million curlers registered in 1200 clubs. In Scotland, there are 50 000 curlers in 52 clubs, and in Sweden there are 9000 curlers in 36 clubs. Write a ratio for each to compare the number of curlers to the number of clubs, and arrange these in order of size from least to greatest.

Strand: Number (Number Operations)
Students will:
- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]

13.4 Using a spreadsheet template and the ratios calculated in 13.3, find the number of curlers for 1 club; i.e., a rate, as well as for 36, 52 and 1200 clubs for each country. Your template should appear similar to the one that follows:

Number of Curlers

Country	1 club	36 clubs	52 clubs	1200 clubs
Sweden		9000		
Scotland			50 000	
Canada				1 000 000

13.5 Have you read, or heard of, the book by Jonathan Swift called *Gulliver's Travels*? Gulliver, a ship captain, suffers a shipwreck and finds himself in the land of Lilliput. Here he finds that the heights of the people, plants and animals are in a 1:12 ratio to the heights of the people, plants and animals in his world. Use a measuring tape to measure yourself. Then complete this chart.

Body Part	Actual Length	Length in Lilliput
Length of arm		
Length of your shoe		
Circumference of your head		

Each day the Emperor of Lilliput gave Gulliver the food and drink necessary to feed about 1 728 Lilliputians. How did the Emperor's mathematicians arrive at this number? Explain why this should be about the right amount.
Note: In this illustrative example, you could use SI or imperial units.

13.6 Which is the better buy:
1.2 L orange juice for $2.50 or 0.75 L orange juice for $1.40?

13.7 Walter and Pat have the same ratio of country CDs to rock CDs. Walter has 3 country CDs for every 5 rock CDs. Pat has 48 country and rock CDs altogether. How many of Pat's CDs are rock?

Strand: Patterns and Relations (Patterns)
Students will:
- use patterns to describe the world and to solve problems.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]

General and Specific Outcomes

General Outcome

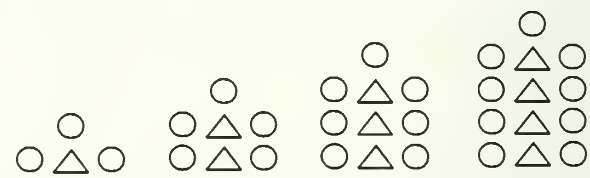
Use patterns, variables and expressions, together with their graphs, to solve problems.

Specific Outcomes

1. Generalize a pattern arising from a problem-solving context, using mathematical expressions and equations, and verify by substitution.
[C, CN, PS, R]

Illustrative Examples [Discretionary]

1.1 Long-Foi made the following pictures with circles and triangles.



He started making a chart to show the number of circles and triangles in each picture.

Picture	Number of Circles	Number of Triangles
1	3	1
2	5	2
3		
4		

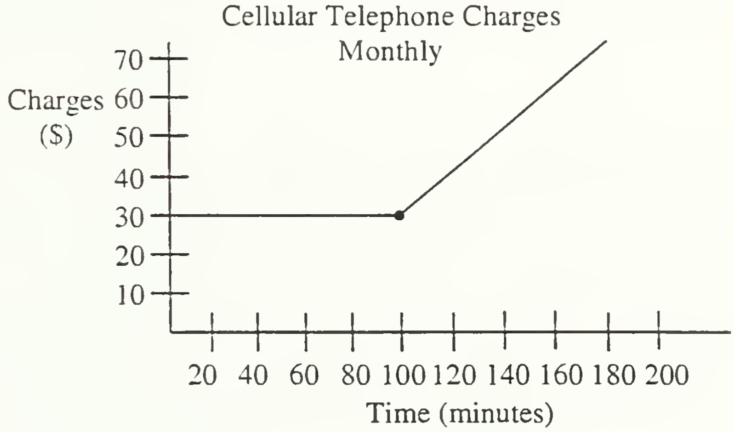
a) Complete Long-Foi's chart and look for a pattern.
b) Write a mathematics sentence to show the relationship between the number of circles and the number of triangles.
c) Make concrete models or pictures to verify your answers.
d) How many circles would you need in a picture with 12 triangles?
e) How can you find and verify the answer?
f) Substitute numbers in your sentence for each picture.

1.2 Read students the story *The King's Chessboard*, by David Birch. Have students explore pattern, by predicting the amounts that will result if one used a doubling pattern to cover the chessboard with grains of rice: one grain in the first square, two grains in the second square, four grains in the third square, and so on. Have them estimate the total number of grains and then find a way to calculate the result.

Strand: Patterns and Relations (Patterns)
Students will:
- use patterns to describe the world and to solve problems.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

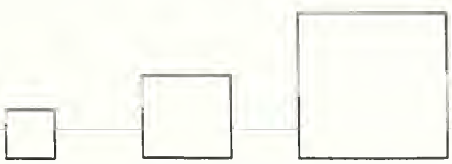
General and Specific Outcomes	Illustrative Examples [Discretionary]
2. Interpolate and extrapolate number values from a given graph. [E, PS, V]	2.1 The graph shows how much Carol pays per month for her cellular telephone use outside of prime-time hours—7 p.m. to 7 a.m.



a) If Carol uses her telephone for 20 minutes in one month, how much will her telephone bill be? How much will her bill be, if she uses her telephone for 85 minutes? Explain.
b) If Carol uses her telephone for 130 minutes, how much will her telephone bill be?
c) If Carol's bill for the month was $65, how many minutes did Carol talk on the telephone?
d) Estimate how much Carol's telephone bill would be, if she used her telephone for 240 minutes in one month. Extend the graph to check your estimate.
e) Explain in words how Carol gets charged by the cellular telephone company.

General and Specific Outcomes	Illustrative Examples [Discretionary]
3. Graph relations, analyze the result and draw a conclusion from a pattern. [R, V]	3.1 Measure the sides of each of the squares provided, and find the perimeter of each square.

a) Make a graph by plotting the length of the sides on the horizontal axis and the perimeters on the vertical axis.
b) Describe the pattern in the graph.
c) From the results of this graph, make a rule for finding the perimeter of a square.
d) Explain how you could verify your rule.



Strand: Patterns and Relations (Patterns)
Students will:
- use patterns to describe the world and to solve problems.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

<table>
<tr><th>General and Specific Outcomes</th><th>Illustrative Examples [Discretionary]</th></tr>
<tr><td>4. Substitute numbers for variables in expressions, and graph and analyze the relation.
[C, PS, R, V]</td><td>4.1 Mitch got a job at “Ease-a-phone”, a new cellular telephone company. He will be paid $20 a day plus $8.50 for every new customer he signs to an “Ease-a-phone” contract.
Mitch used the expression E = 20 + 8.5C to determine his earnings per day—E stands for earnings, and C stands for new contracts.

C | 0 | 1 | 2 | . . .
E | 20 | 28.50 | 37.00 | . . .

a) Which variable is dependent? On which axis will it go?
b) Continue Mitch’s chart and make a graph to show the relationship.
c) Will you join the points on the graph with a line? Explain.
d) Discuss the pros and cons to this kind of earnings arrangement.</td></tr>
<tr><td>5. Translate between an oral or written expression and an equivalent algebraic expression.
[C, CN]</td><td>5.1 Write an algebraic expression for the following:
Force equals mass times acceleration.

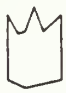
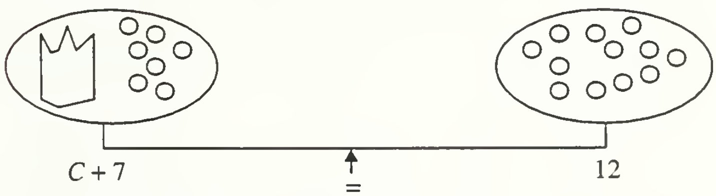
5.2 Describe the following algebraic equations in words.

a) $A = \frac{1}{2}bh$
b) $P = 2l + 2w$

5.3 Carl has 30 coins, all dimes and quarters. Write an expression to represent the number of dimes, if he has x quarters.</td></tr>
</table>

Strand: Patterns and Relations (Variables and Equations)
Students will:
- represent algebraic expressions in multiple ways.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Use variables and equations to express, summarize and apply relationships as problem-solving tools. **Specific Outcomes**	
6. Evaluate expressions with and without concrete models. [R, V]	6.1 An expression for the mass of two cans and five bearings is $2c + 5b$. Find the total mass, if each can has a mass of 200 g and each bearing has a mass of 75 g. 6.2 A formula for finding the perimeter of a rectangle is $P = 2l + 2w$. Find the perimeter when l is 8 cm and w is 6 cm.
7. Solve and verify one-step linear equations, using a variety of techniques including concrete materials and diagrams. [PS, R]	7.1 Have students use manipulatives or models to explore the concept of balance or equality; e.g., have students use: a) a weight balance b) hands-on equations. Adapted with permission of the British Columbia Ministry of Education. 7.1.1 Jill had some CDs in a case. After Jay gave her 7 more, she had 12 CDs in all. How many CDs were in the case? Jose wrote the equation $C + 7 = 12$. He used a balance scale model to solve it. Let  represent the CDs Jill originally had.  He took 7 CDs off each side. 
	Illustrate Jose's method in solving the following problem: Bill had six books. Teruko gave him some more, and then he had 10 books. How many books did Teruko give Bill?

Strand: Patterns and Relations (Variables and Equations)
Students will:
- represent algebraic expressions in multiple ways.

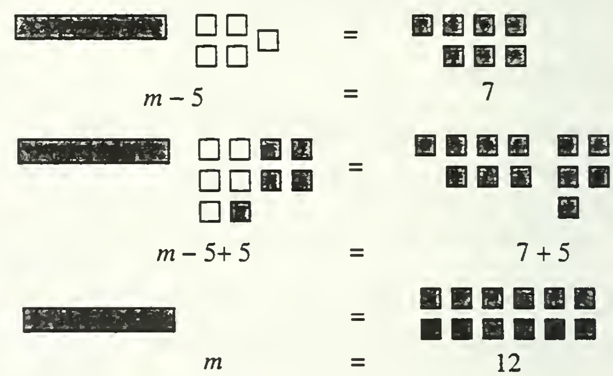
[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]

Illustrative Examples [Discretionary]

7.1.2

Sharon had some money; she spent $5 and then she had $7 left. How much money did Sharon have to begin with?
Ted wrote the equation $m - 5 = 7$ and used algebra tiles to solve it.



Sharon had $12 to begin with.

Use Ted's method to solve:
Barb had some sports cards. She sold six and then she had 10 left. How many did she have to start with?

7.2 Have students solve equations, showing all their work. Ask them to exchange solutions with other students and use keys to mark each other's work. Students can identify the errors in their peers' solutions and explain how to fix them, and they can use partner feedback to correct their own work.

Adapted with permission of the British Columbia Ministry of Education.

7.3 Discuss with students the value of algebraic operations for solving problems. Have them consider longer, more complicated equations that can be solved easily and quickly using algebra but that take much longer to solve using arithmetic.

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General and Specific Outcomes

8. Solve and verify two-step, single-variable, first-degree equations, using concrete materials, diagrams, informal algebraic methods, or formal algebra in the form:
   - $x + a = b$
   - $ax = b$
   - $\frac{x}{a} = b$
   - $ax + b = c$

   where a, b and c are integers.
   [CN, PS, V]

Illustrative Examples [Discretionary]

8.1 Nat had some stamps. He divided them into sets of six to put in his album. He made 17 sets. How many stamps did Nat have?

8.2 Joe had 5 sports cards. He bought 3 packs with the same number of cards in each pack. If he now has 35 cards in all, how many were in each pack? Solve, by using algebra tiles. Write an equation you could use to solve this problem.

Strand: Patterns and Relations (Variables and Equations)
Students will:
- represent algebraic expressions in multiple ways.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
	8.3 Hans gave Vera half his raffle tickets to sell. She lost seven of the tickets Hans gave her and had 23 left. How many tickets did Hans have to start with? Write an equation, and show how to solve it algebraically. Verify your answer by substituting it in your equation or by using counters.
	8.4 Suggest that students work in pairs to construct flow charts of equation-solving procedures, then switch with other pairs and attempt to apply the steps to a given problem. Is there more than one way to solve the problem? Discuss as a class. Reproduced with permission of the British Columbia Ministry of Education.
	8.5 To start a summer yard work program, a student took out a \$3000 loan for 3 months. The student paid \$90 interest on the loan. Find the monthly interest rate, using the formula $I = p \cdot r \cdot t$ (Interest = Principal × Interest Rate × Time).
	8.6 Kassidy bought five CDs at the same price each and paid a total of \$84.45. How much did each CD cost? Write an equation, and show how to solve it algebraically. Verify your answer by substituting it in your equation.
	8.7 Maria had a length of fabric to make banners. She divided the fabric into six equal pieces, and each piece was 2.75 m long. a) What was the length of the fabric? Write an equation, and show how to solve it algebraically. b) Verify your answer by substituting it in your equation or by using strips of grid paper.
9. Create and solve problems, using first-degree equations. [PS]	9.1 The following is some information that will be the basis of your constructing a word problem. It is 300 km from Regina to Gull Lake, Saskatchewan. About halfway between the two locations is Chaplin. Chelsea drives her car the speed limit on No. 1 highway. Alain drives his convertible 10 km slower than Chelsea. Write two problems or questions based on this information.

Strand: Shape and Space (Measurement)
Students will:
- describe and compare everyday phenomena, using either direct or indirect measurement.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Estimate, measure and compare, using decimal numbers or fractions and standard imperial and SI units of measure. **Specific Outcomes**	
1. Estimate, measure, record, compare and order objects by length, height, perimeter and circumference, using standard imperial and SI units. [E, PS]	1.1 Use a micrometer to measure the thickness of 10 sheets of paper. Use the results of this measurement to determine the thickness of one sheet of paper. 1.2 Calculate the area of a flat rectangular surface measuring 21 feet by 14 feet.
2. Estimate, measure, record and compare the volume/capacity of containers, using standard imperial and SI units. [E, PS]	2.1 Estimate the volume of a water bed bladder having a depth of 12 inches, a width of 5½ feet and a length of 80 inches. 2.2 Measure the internal dimensions of a rectangular container, and calculate its volume in cm^3. Find its volume in litres, and verify using a calibrated cylinder.
3. Estimate, measure, record, compare and order the weight and mass of objects, using standard imperial and SI units. [E, PS]	3.1 Fill three different empty tin cans with sand; e.g., a drink can, a soup can and a tuna can. Estimate the weight/mass of each. Use standard weights/masses and a balance scale to check your estimates. 3.2 Ask students to find objects that have a mass that is less than, more than and equal to 1 kilogram. Ask them to find objects that have a weight that is less than, more than and equal to 1 pound. Have them measure, record, compare and order the masses and weights. (This could be expanded to a scavenger hunt for objects of any given measurement.) 3.3 Ask students to construct an object, using modelling clay, that is a pound/kilogram. Have them measure to determine accuracy and share their strategies for determining the weight/mass of their modelling clay object.
4. Estimate, read and record temperature to the nearest degree Fahrenheit or degree Celsius. [E]	4.1 Study the daily temperatures in a community for a week. Identify the daily high and low temperatures, and graph the daily highs, daily lows and daily differences. Have students work in pairs to share their data and pose questions to each other related to their findings. 4.2 Read and record the temperatures on thermometers placed in various locations; e.g., in the shade, in the Sun, beside a window or door. Compare the temperatures. What might account for any differences?

Strand: Shape and Space (Measurement)
Students will:
- describe and compare everyday phenomena, using either direct or indirect measurement.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
	4.3 Read and record the temperatures of various objects, using both Fahrenheit and Celsius units of measure.
5. Convert between SI and imperial systems of measure, using a conversion table or calculator. [CN, PS, R, T]	5.1 Metric Conversion Table (see below)

5.1 Metric Conversion Table

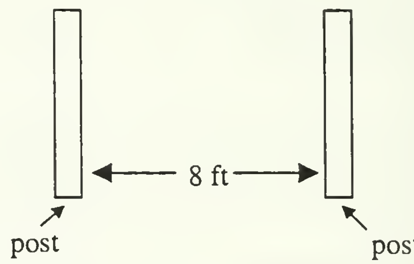
Capacity:	
1 Canadian gallon	4.546 litres
1 litre	0.220 Canadian gallons
Volume:	
1 cubic inch	16.387 cubic centimetres
1 cubic centimetre	0.061 cubic inches
1 cubic yard	0.765 cubic metres
1 cubic metre	1.308 cubic yards
Length:	
1 inch	2.540 centimetres
1 centimetre	0.394 inches
1 foot	0.305 metres
1 metre	3.281 feet
1 yard	0.914 metres
1 metre	1.094 yards
1 mile	1.609 kilometres
1 kilometre	0.621 miles
Area:	
1 square inch	6.452 square centimetres
1 square centimetre	0.155 square inches
1 square foot	0.093 square metres
1 square metre	10.764 square feet
1 square yard	0.836 square metres
1 square metre	1.196 square yards
1 acre	0.405 hectares
1 hectare	2.471 acres
Mass:	
1 ounce	28.350 grams
1 gram	0.035 ounces
1 pound	0.454 kilograms
1 kilogram	2.205 pounds

5.1.1 Bill's boss wants him to buy 600 pounds of concrete. Bill goes to "Builders' World" and notices that concrete can be purchased in 22 kg bags. How many bags should he buy?

5.1.2 Ryan is ordering gravel for a garage and driveway and determines he needs $8m^3$ of gravel. He telephones a gravel delivery company and finds out he can only order by the cubic yard. How many cubic yards of gravel should Ryan order?

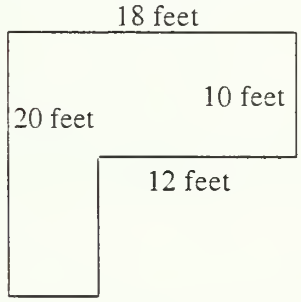
Strand: Shape and Space (Measurement)
Students will:
- describe and compare everyday phenomena, using either direct or indirect measurement.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
6. Use conversions among commonly-used, standard units of length to solve problems. [CN, PS, R, T]	6.1 Frank is building a fence with 8 foot sections. Assuming there is to be no space between the boards, how many 1x6 fence boards should he order for each section? (Investigate the actual dimensions of a 1x6 fence board.)  Frank's fence has a total of 12 sections. How many boards should he order, if he includes 3% extra to allow for unusable lumber?
General Outcome Generalize measurement patterns and procedures, and solve problems involving area, perimeter, surface area and volume. **Specific Outcomes** 7. Develop, verify and use rules or expressions for determining the areas and perimeters of polygons. [C, CN, PS, T]	7.1 The dimensions of five decorative gardens are given below. Which garden has the greatest area? a) square with sides 10.2 m b) rectangle with length 15 m and width 6.9 m c) parallelogram with base 14.6 m and height 7.2 m d) triangle with base 16.5 m and height 12.4 m e) trapezoid with bases of 18.1 m and 10.4 m, and height 7.1 m.

Strand: Shape and Space (Measurement)
Students will:
- describe and compare everyday phenomena, using either direct or indirect measurement.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

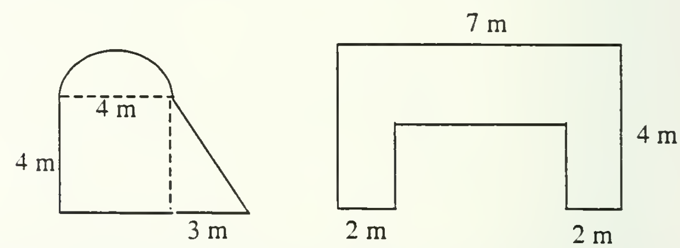
General and Specific Outcomes	Illustrative Examples [Discretionary]
	7.2 Silken, a journeyman carpenter, is installing hardwood flooring in her friend's L-shaped living room. She makes the following sketch of the room and includes measurements.  a) How many square feet of floor need to be covered? b) One package of hardwood flooring will cover 14 square feet and costs \$42.00 per package (not including GST). Silken knows that she should purchase 10% more material to allow for waste. • How many packages of hardwood flooring should she buy? • How much will the friend pay for hardwood alone? c) How much will Silken be paid if she charges her friend \$2.50 per square foot of area covered? 7.3 You want to paint one wall of your room. The wall is 9.7 m long and 6.8 m high. It takes one litre of paint to cover 10 m^2, and the paint sells for \$9.97 a litre. a) What would it cost you, if you purchase only paint? b) What else do you need to think of? 7.4 Melodie said that to find the perimeter of a triangle, you only have to measure one side and multiply by 3. Do you agree? Cut straws in several different lengths and make as many different triangles as you can. Use these straw triangles to explain your answer. Make a rule to find the perimeter of a triangle.

Strand: Shape and Space (Measurement)
Students will:
- describe and compare everyday phenomena, using either direct or indirect measurement.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]

7.5 Ask students to estimate the perimeters and surface areas of composite shapes and objects, like the ones below, then measure, calculate and compare the derived values with their estimates. Have students use their journals to reflect on the differences between their estimates and the actual measurements. Students could use drafting software to check their estimates.



Adapted with permission of the British Columbia Ministry of Education.

7.6 Aaron sketched some parallelograms on grid paper and cut them out. Then he cut a piece off one end of each parallelogram and fit it on the other side to form a rectangle. He made this chart:

Base of Parallelogram	Height of Parallelogram	Area of Parallelogram	Base of Rectangle	Height of Rectangle	Area of Rectangle
3	4	12	3	4	12
2.5	3.5	8.75	2.5	3.5	8.75
1.5	4.2				
3	6.5				

Finish Aaron's chart, and look for a pattern. Test your pattern. Make a rule to find the area of a parallelogram. What other information should Aaron include on his chart to identify a pattern for finding the perimeter of a parallelogram?

Strand: Shape and Space (Measurement)
Students will:
- describe and compare everyday phenomena, using either direct or indirect measurement.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]

7.7 Have students create simple blueprints of their dream homes, specifying the measurements of each room and calculating the areas and perimeters. Can students explain why it is important to know these measurements if they are going to build a house? Ask students to exchange their projects with other students and evaluate each other's work, using the following criteria:

a) Are the measurements realistic?
b) Based on the measurements students have supplied, are the calculations of area and perimeter accurate?

Collect and review the students' blueprints and the corrections and comments made by peers, and provide feedback.

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7.8 Give pairs of students maps of the world and select five countries for the class to study. Have each pair of students estimate the area of each country, using the map's scale and a centimetre grid. When they have finished, have them collect the class data and calculate an average area for each country. Have students consult the social studies text or another reference to check if the average determined by the class or an estimate determined by one of the pairs is closest to the actual area of each country. Encourage students to express the area in various ways; e.g., 3 400 000 km^2 or 3.4 million km^2.

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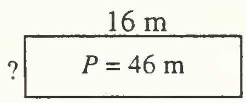
7.9 Domenico has forgotten one dimension of a room. What is the missing dimension.



Three students each began to solve the problem.

Domenico wrote: $P = L + W + L + W$
$46 = 16 + W + 16 + W$

Jake wrote: $P = (2 \times L) + (2 \times W)$
$46 = (2 \times 16) + (2 \times W)$

Owen wrote: $P = 2\,(L + W)$
$46 = 2\,(16 + W)$

Choose one boy's work and finish solving the problem. What is the missing dimension? Explain why you chose the work you did.

Strand: Shape and Space (Measurement)
Students will:
- describe and compare everyday phenomena, using either direct or indirect measurement.

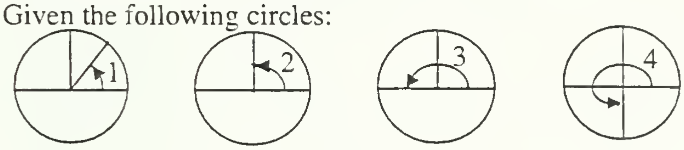
[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
8. Describe the effects of dimension changes in related 2-D shapes when solving problems involving area. [C, CN, PS, R, V]	8.1 Have students use pipe cleaners or other construction materials to determine if a constant perimeter gives a constant area. Reproduced with permission of the British Columbia Ministry of Education. 8.2 Barrie wanted to fence a rectangular garden area. The fencing material comes in 1-m lengths that cannot be cut. If Barrie has 12 m of fencing, what are the dimensions of the largest garden area he can make? Draw a diagram to explain your reasoning.
General Outcome Solve problems involving perimeter, area, surface area, volume and angle measurement. **Specific Outcomes** 9. Estimate, measure and draw angles, using a protractor. [E]	9.1 You are to draw a map to be used on a class field trip. Use a 8½ x 11 inch sheet of white paper and label the top north. You might want to indicate south, east and west also. a) Mark and label point A as a starting point half way across the page, about 4 cm from the top. b) Make a line straight south 4 cm to point B. Draw a picnic table beside this point. c) From point B, measure a 150° angle going in a SE direction to form ∠ABC. Make line BC 8 cm long, and draw a garbage can beside this end point. d) From point C, measure a 225° angle going in a SW direction to form ∠BCD. Make line CD 11.5 cm long. This point is on the shore of the lake. Draw the lake. e) From point D, make a 7 cm line going straight west to point E. Draw the pier here. f) From point E, measure a 75° angle going in a NE direction to form ∠DEF. Make line EF 9 cm long. Draw a wood pile beside the end point. g) From point F, measure a 90° angle going west to form ∠EFG. Make line FG 6 cm, and draw a campground symbol here. h) From point G, measure a 60° angle going NE to form ∠FGB. You should be back at the picnic table. If the scale for your drawing is 1 cm = 0.25 km, how long is the trail you drew?

Strand: Shape and Space (Measurement)
Students will:
- describe and compare everyday phenomena, using either direct or indirect measurement.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

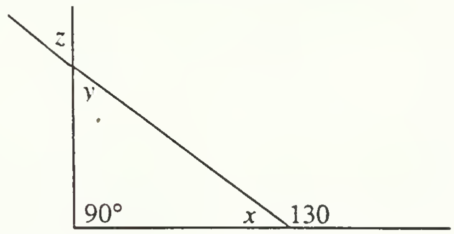
General and Specific Outcomes	Illustrative Examples [Discretionary]
	9.2 Given the following circles:



a) Use a protractor to measure the above angles: $\angle 1, \angle 2, \angle 3, \angle 4$.
b) Indicate the portion of the circle that each angle corresponds to; e.g., $\angle 3 = \frac{1}{2}$ circle.
c) If you had a circle divided into 6 equal parts through the centre, what would be the measure of each sector in degrees?

10. Use mathematical reasoning to determine the measures of angles in a diagram.
[R, V]

10.1 Find the measures of the indicated angles in the diagram to the right.



General Outcome

Solve problems involving the properties of circles and their connections with angles and time zones.

Specific Outcomes

11. Measure the diameters, radii and circumferences of circles, and establish the relationships among them.
[CN, R]

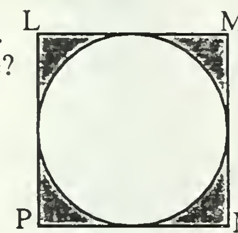
11.1 Gunther gathered a variety of circular objects, such as container lids and wheels. For each object, he measured the diameter with calipers and the circumference with a tape measure. He started making this chart:

Object	Diameter	Circumference	Relationship between Diameter and Circumference
nut can lid			
bicycle wheel			
coffee can			

He noticed a pattern in how the two measures for each object were related.
a) Estimate the relationship between diameter and circumference. Test it by measuring the diameter of another object and predicting the circumference before measuring it.
b) Use your calculator to find the relationship in each case.
c) Make a rule that relates the diameter and circumference of a circle.

Strand: Shape and Space (Measurement)
Students will:
- describe and compare everyday phenomena, using either direct or indirect measurement.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
	11.2 Have students compare the difference in area of small, medium and large circular pizzas. Ask them to construct a graph to represent the relationship between the diameter and the area for each size. What generalizations can they make for this relationship? Reproduced with permission of the British Columbia Ministry of Education.
12. Solve problems involving the radii, diameters and circumferences of circles. [PS, T]	12.1 The pointed end on the minute hand of a clock travels 132 cm each hour. How long is the minute hand? 12.2 If Nel's bicycle wheel is 28 inches in diameter, how far can Nel go in one revolution? How far can she go in 10 revolutions? 12.3 The perimeter of the square *LMNP* is 60 cm. What is the length of each side of the square? What is the area of the square? Find the: a) diameter of the circle b) circumference of the circle c) area of the circle d) area of the shaded region. 
General Outcome Apply indirect measurement procedures to solve problems. **Specific Outcomes** 13. Use concrete materials and diagrams to verify the Pythagorean relationship. [CN, R]	13.1 Tara is investigating the relationship among the three sides of a right triangle. She drew a right triangle in the middle of a sheet of grid paper and then constructed a square on each side of the triangle. Then she tried to cut the two smaller squares and fit them on the largest square. Try Tara's investigation, using right triangles with different shapes. Explain what you find.

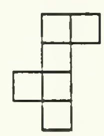
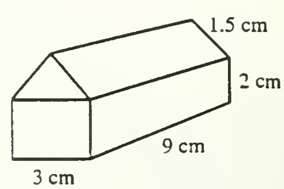
Strand: Shape and Space (Measurement)
Students will:
- describe and compare everyday phenomena, using either direct or indirect measurement.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
	13.2 Divide a piece of rope or string into 12 equal lengths, by placing tape on 13 spots. Ask three students to hold the rope in a way that constructs a triangle with a right angle—each student represents a vertex; one student holds the two ends of the rope together. Students should discover that there is only one solution. Discuss: a) Can any other right triangle be made? b) Does 3, 4, 5 always result in a right triangle? c) What are the applications? Challenge students to design another way to divide a rope that produces a right triangle; e.g., 5, 12, 13=30 parts; 7, 24, 25=56 parts. Students could use a calculator to find the triples. Adapted with permission of the British Columbia Ministry of Education.
	13.3 Ask students to generate their own charts of Pythagorean triples and their multiples. For example: Original Triple / Times 2 / Times 3 / Times 10 3, 4, 5 / 6, 8, 10 / ___, ___, ___ / ___, ___, ___ 5, 12, 13 / ___, ___, ___ / 15, 36, 39 / ___, ___, ___ 7, 24, 25 / ___, ___, ___ / ___, ___, ___ / 70, 240, 250 Reproduced with permission of the British Columbia Ministry of Education.
14. Use the Pythagorean relationship to calculate the measure of the third side, of a right triangle, given the other two sides in 2-D applications. [PS]	14.1 Jamie wants to walk from one corner of the rectangular playing field to the opposite corner. The playing field is 90 feet by 150 feet. What is the shortest route he can take? Explain. 14.2 Invite students to find right triangles in the classroom. For each triangle, have students measure two of the sides and then use the Pythagorean relationship to calculate the third side. Are their calculations accurate? Can they explain the process? Reproduced with permission of the British Columbia Ministry of Education. 14.3 Provide pairs of students with a list of things to measure using the Pythagorean relationship. Ask students to compare their measurements in class and discuss the reasons for possible differences. Are the measurements reasonable? How plausible are their explanations of possible reasons for differences? Reproduced with permission of the British Columbia Ministry of Education.

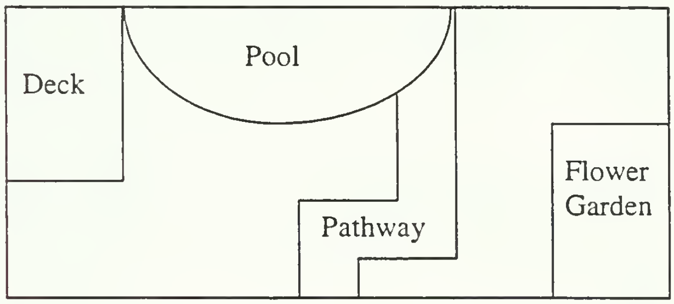
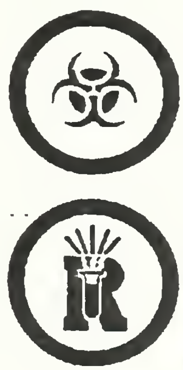
Strand: Shape and Space (3-D Objects and 2-D Shapes)
Students will:
- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Use spatial problem solving in building, describing and analyzing geometric shapes. **Specific Outcomes**	
15. Build 3-D objects from a variety of representations (nets, skeletons). [PS, V]	15.1 Raymond cut this net for a cube from grid paper. How many different nets can you cut that make cubes?  15.2 Find two different nets for a cylinder. 15.3 Use toothpicks and molding clay to build prisms and pyramids with various polygons for bases.
16. Draw the plan and elevations of a 3-D object from sketches and models. [C, R, T, V]	16.1 Draw and label the plan and the front, right and left elevations of this sketch. 

Strand: Shape and Space (3-D Objects and 2-D Shapes)
Students will:
- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Create and analyze design problems and architectural patterns, using the properties of scaling, proportion and networks. **Specific Outcomes**	
17. Represent, analyze and describe enlargements and reductions. [CN, R]	17.1 Describe some everyday situations in which 2-D enlargements and reductions are necessary or useful; e.g., photocopies, photographs, scale models, statues. Explain how the enlargement or reduction is the same and how it is different from the original figure or object; e.g., size, shape, proportion. 17.2 If the following is drawn on 1 cm grid paper, draw its enlargement on 2 cm grid paper.  17.3 Using the following WHMIS symbols, have students place the original over 1 cm by 1 cm grid paper and then enlarge it on 3 cm by 3 cm grid paper to make a class poster. 

Strand: Statistics and Probability (Data Analysis)
Students will:
- collect, display and analyze data to make predictions about a population.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Evaluate and use measures of central tendency and variability. **Specific Outcomes**	
1. Determine measures of central tendency and variability for a set of data: • mode • median • mean • range. [PS]	1.1 You are the statistician for the school football team and have been asked by the yearbook editor for the mean height of the 45 team members. You collect the heights of the 45 players. They vary from 5 feet 6 inches to 6 feet 8 inches, and are each measured to the nearest half-inch. Using a calculator or a computer: a) describe in detail the keying procedures or menu choices that can be used to determine the mean height of the 45 players b) show that your procedure works, by using it to find the mean of 5 feet 11 inches, 6 feet 2 ½ inches, 5 feet 9 inches, 6 feet 5 inches, and 5 feet 7 ½ inches. 1.2 Do a survey of your class on favourite car colour, and determine the mode.
2. Determine and use the most appropriate measure of central tendency in a given context. [CN, PS, T]	2.1 Explain why each of the following people might select the mean, median or mode in a set of data. a) A store owner deciding what sizes of shoes to order. b) Someone moving to a new city and looking at housing costs. c) Reporting the average score on a test.

Strand: Statistics and Probability (Data Analysis)
Students will:
- collect, display and analyze data to make predictions about a population.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]

General Outcome

Develop and implement a plan for the collection, display and analysis of data using technology, as required.

Specific Outcomes

3. Read and interpret graphs.
   [C, E, PS, R]

3.1 Find some graphs in your local newspaper. Working in a group, discuss the following questions.
   a) Does the data give the information indicated?
   b) Is the type of graph appropriate for the data?
   c) Have scales, pictures or data groupings been selected to distort data?
   d) Does the graph present data clearly? Is it visually appealing?

4. Discuss how collected data are affected by the nature of the sample, the method of collection, the sample size and biases.
   [C, CN]

4.1 Carmen designed and handed out 100 questionnaires to middle year students in her school. One question she asked was this:

What do you want to be? Choose one.

☐ Doctor/Dentist ☐ Teacher
☐ Lawyer ☐ Sports Manager, Coach or Trainer

50 questionnaires were returned. Here are the results.

	Boys	Girls
Doctor/Dentist	𝍸 𝍸	𝍸 |
Teacher	| |	𝍸 𝍸 |
Lawyer	𝍸 | | |	𝍸 | |
Sports Manager, etc.	𝍸	|

Carmen reached the conclusion that most students will become doctors or dentists.

Tell if you agree with each of the following, and describe what else Carmen might have done:
a) the wording of Carmen's question
b) the method of gathering data
c) the sample she chose to survey
d) the conclusion she reached.

Strand: Statistics and Probability (Data Analysis)
Students will:
- collect, display and analyze data to make predictions about a population.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
5. Describe issues to be considered when collecting data; e.g., appropriate language, ethics, cost, privacy, cultural sensitivity. [C, CN, R]	5.1 For each of these questions: a) Is there a relationship between wrist circumference and height? b) Does smoking cause lung cancer? c) Does pet ownership enhance the quality of life for senior citizens? Explain what would be the most appropriate methods for collecting data. Identify potential ethical problems, need for sensitivity to personal and cultural beliefs, and cost when designing questions and collecting data.
6. Select, defend and use appropriate methods of collecting data: • designing and using questionnaires • interviews • experiments • research. [C, PS, T]	6.1 As a member of your school's student council, you wish to determine the number of students who will attend a school dance. Explain how you could collect this information. 6.2 A city council is trying to determine if a major road in your district needs to be widened. Explain a procedure that could be used to help city council make this decision. 6.3 Bill surveyed people entering a theatre and asked them if they would support building a new sports arena in the community. Discuss this method of collecting data in terms of its appropriateness.
7. Make predictions from data. [E, PS, R]	7.1 Collect data on the population of your school over the last 10 years. Graph the information. What would you predict the school population will be in six years? What trends do you see in the population? Can you suggest any reasons for these trends? Would enrollment figures suggest a need to increase the size of your school facility in the next ten years? Why or why not?

Strand: Statistics and Probability (Data Analysis)
Students will:
- collect, display and analyze data to make predictions about a population.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
8. Display data by hand or by computer in a variety of ways, including circle graphs. [C, T, V]	8.1 Have students work in groups to research, via the Internet, examples of statistics used in a variety of situations; e.g., population statistics, professional sport, earthquake prediction, weather, loan risk. Students report their findings to the class, using appropriate graphs to display their data. Adapted with permission of the British Columbia Ministry of Education. 8.2 Keep a record of all your activities in a typical school day (24 h). Decide on categories for which the activities can be reported in number of hours (sleeping is an activity). Make a circle graph to show your typical school day. Share and compare graphs with other students. How can someone else's graph be useful to you?
9. Critique ways in which statistical information and conclusions are presented by the media and other sources. [C, CN]	9.1 Collect data presented via newspaper, magazine, radio or television. a) How were samples for the data selected? Why do you think they were selected that way? Are they biased? b) Were the data collection methods appropriate for the data and the issue? c) How would you do it differently? Why? d) Are the data presented clearly and honestly? e) Do the conclusions follow logically from the data? f) What questions are left unanswered? Is this deliberate?

Strand: Statistics and Probability (Chance and Uncertainty)
Students will:
- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Compare theoretical and experimental probability of independent events. **Specific Outcomes**	
10. Compare experimental results with theoretical results. [C, E, R]	10.1 You have a cube with faces numbered 1 to 6. a) What is the theoretical probability of rolling: a 6? a 4? a 1? b) Perform an experiment with a die and compare the results. 10.2 Ask students to work in groups to develop hypotheses that they can test; e.g., probability of selecting a given coloured candy from a bag when the quantities of each colour are known. Have groups calculate the theoretical probabilities, then conduct experiments to test their hypotheses. Reproduced with permission of the British Columbia Ministry of Education. 10.3 Invite an insurance agent to discuss with the class how insurance companies incorporate independent events in the calculation of premiums. Reproduced with permission of the British Columbia Ministry of Education.
11. Calculate theoretical probability, using numbers between 0 and 1. [E, PS]	11.1 There are 350 door prize tickets sold. If you buy 5 tickets, what is the probability of you winning the 1st prize drawn.
12. Recognize that if n events are equally likely the probability of any one of them occurring is $\frac{1}{n}$. [R]	12.1 If you toss one standard die, what are the possible outcomes? Are they equally likely? Explain. Write the probability of tossing a 4. If you did the same experiment with a 12-sided die, what would be the probability of tossing a 4? 12.2 If you draw a card from a deck, what suit could it be? Are all suits equally likely? What is the probability of drawing a heart?

Strand: Number (Number Operations)
Students will:
- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Describe and apply arithmetic operations on tables to solve problems, using technology as required. **Specific Outcomes** 1. (A2–1) Solve problems involving combinations of tables, using: • addition or subtraction within tables • multiplication of a table by a real number • spreadsheet templates. [PS, T, V]	1.1 The following is an income and expenses report for a business for the year ending December 31.

	Year 1	Year 2	Year 3
Total Sales	$135 000	$148 000	$166 000
Expenses			
Accounting	$ 1 500	$ 1 600	$ 1 900
Advertising	2 000	2 000	3 000
Rent	12 000	14 000	16 000
Repairs	4 000	5 000	5 500
Salaries	74 000	79 000	81 000
Miscellaneous	10 000	13 500	9 000
Total Expenses	$	$	$
Profit Before Tax	$	$	$
Income Tax (25%)	$	$	$
Net Profit	$	$	$

Enter the data from above onto a spreadsheet template provided to students. The template should include formulas to calculate missing values.

1.1.1 a) Calculate the dollar change in total sales and total expenses between each year in the table.
b) What are the reasons for the change in profit?

(continued)

Strand: Number (Number Operations)
Students will:
- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

<table>
<tr><th>General and Specific Outcomes</th><th>Illustrative Examples [Discretionary]</th></tr>
<tr><td></td><td>
1.1 (continued)

1.1.2 a) Calculate the percentage change in total sales and total expenses between each year in the table.
b) Do your results agree with your answer in 1.1.1(b) above? Explain.

1.1.3 Prepare a line graph showing total sales, total expenses and profit before tax for the three-year period. Use the graph to determine which item has increased the most. Does your graph agree with your previous results?

1.1.4 Prepare a column for Year 4, assuming a 3 per cent increase in sales from Year 3. Generate your own expense column, and justify changes.

1.2 It costs $1.37 Canadian to buy 1 US dollar. Note: This rate changes daily.

a) Find the ratio of a Canadian dollar to a US dollar.
b) Find the ratio of a US dollar to a Canadian dollar.
c) How many US dollars will it take to buy 1 Canadian dollar?

These rates are located in currency tables and can be used to convert between different currencies.

<table>
<tr><th></th><th>Canadian dollar</th><th>US dollar</th></tr>
<tr><td>Canadian dollar</td><td>—</td><td>1.37</td></tr>
<tr><td>US dollar</td><td>0.73</td><td>—</td></tr>
</table>
d) If you want to buy 500 US dollars, how many Canadian dollars would you have to spend?
e) Would it cost the same amount to buy $500 US in traveller's cheques? Explain.
f) Investigate commission rates charged by local financial institutions on money exchange transactions.
</td></tr>
</table>

Strand: Number (Number Operations)
Students will:
- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
	See below

1.3 A banker needs to provide clients with information on foreign exchange. Use the foreign exchange chart provided, or a current chart from a newspaper, to answer the following questions.
   a) Calculate the cost in Canadian dollars of a refrigerator that costs $850 US.
   b) Calculate the cost in US dollars of an outboard motor selling in Canada for $1200.
   c) Hans receives a cheque for 100 Swiss francs from his uncle in Berne. How many Dutch guilders would he get for this cheque? How many Canadian dollars?

Foreign Exchange
Cross Rates

	Canadian dollar	US dollar	British pound	German mark	Japanese Yen
Canada dollar	–	1.3743	2.0762	0.9227	0.012850
US dollar	0.7276	–	1.5107	0.6714	0.009350
British pound	0.4816	0.6619	–	0.4444	0.006189
German mark	1.0838	1.4894	2.2501	–	0.013927
Japanese yen	77.82	106.95	161.57	71.81	–
Swiss franc	0.8821	1.2122	1.8313	0.8139	0.011335
French franc	3.7230	5.1165	7.7297	3.4352	0.047841
Dutch guilder	1.2134	1.6676	2.5194	1.1196	0.015593
Italian lira	1156.07	1588.79	2400.23	1066.71	14.855491

Foreign Exchange
Cross Rates

	Swiss franc	French franc	Dutch Guilder	Italian lira
Canada dollar	1.1337	0.2686	0.8241	0.000865
US dollar	0.8249	0.1954	0.5997	0.000629
British pound	0.5460	0.1294	0.3969	0.000417
German mark	1.2287	0.2911	0.8931	0.000937
Japanese yen	88.23	20.90	64.13	0.067315
Swiss franc	–	0.2369	0.7269	0.000763
French franc	4.2208	–	3.0681	0.003220
Dutch guilder	1.3757	0.3259	–	0.001050
Italian lira	1310.64	310.52	952.72	–

Strand: Number (Number Operations)
Students will:
- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Solve consumer problems, using arithmetic operations. **Specific Outcomes**	
2. (C4–1) Solve consumer problems, such as: • wages earned in various situations • property taxation • exchange rates • unit prices • personal income tax calculation. [CN, E, PS, R, T]	2.1 Calculate and compare wage situations involving minimum wage rates, regular pay, overtime pay, gratuities, piecework, straight commission, salary and commission, salary plus graduated commission. 2.2 Jane has a choice of two restaurants at which to work. Mario's pays \$8/h, and tips average \$24 daily. Teppan's pays \$5.50/h, and tips average \$35 daily. If Jane works 30 hours weekly, spread over four days, how much would she earn at each restaurant? 2.3 Identify and calculate various payroll deductions, including income tax, CPP, UI, medical benefits, union and professional dues and life insurance premiums. 2.4 You can buy 900 g of dog food for \$2.69 or 12.5 kg for \$37.15. Which is the better buy? 2.5 Calculate your personal income tax, using an appropriate printed or electronic Revenue Canada Taxation form. **Note:** Revenue Canada will supply student activity booklets for this.

Strand: Number (Number Operations)
Students will:
- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

<table>
<tr><th>General and Specific Outcomes</th><th>Illustrative Examples [Discretionary]</th></tr>
<tr><td>3. (C4–2) Reconcile financial statements, such as:
• cheque books and electronic bank transactions with bank statements
• credit card statements with payment receipts.
[CN, PS, T]</td><td>3.1 The March opening balance of your bank account is $2504.32. The following personal transactions occurred during the month.

March 1 pre-authorized withdrawal of $95.92 for car insurance
March 1 interest payment of $0.04
March 4 deposited pay cheque of $150.00
March 4 withdrew $20.00 for gas
March 10 paid $8.72 for lunch, using debit card
March 12 paid $17.11, by cheque, for a CD
March 15 deposited pay cheque of $150.00
March 17 paid $42.76 for a concert ticket, using debit card
March 21 withdrew $25.00 spending money
March 26 paid $21.87 for gas, using debit card
March 31 transaction fee of $2.00

a) Determine the total amount of credits.
b) Determine the total amount of debits.
c) Find your bank balance for the end of March.
d) Describe the pros and cons of paying a flat rate transaction fee.

3.2 Fill in the missing information:
Credit Card Statement Account #123456

09/07 Previous Balance Owing $125.00
Payment Thank you 100.00
11/07 Ralph's Tire Shop 36.99
20/07 The Shoe Shop 27.99 cr
27/07 Super Sports 24.87
30/07 Disco Jim's 79.99</td></tr>
</table>

Maximum Credit	Balance on Last Statement	Interest Charged	Total Purchases This Statement	Total Payment and Credits This Statement	New Balance
$500.00	$125.00	$2.30			
Due date: August 22			Minimum Payment 10% of Balance		
			Credit Remaining		

Strand: Number (Number Operations)
Students will:
- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
	3.3 Complete the table below to determine the cost of credit for using a charge account for the period shown. Monthly credit charges are 1.4% of the balance due.

Month	Previous Balance	–	Payment Made	+	Purchases Charged	=	Balance Due	+	Credit Charges	=	New Balance
February	$314.65		$100.00		$193.75				$5.72		$414.12
March			$150.00		$ 59.60						
April			$140.00		$421.83						$618.62
May	$618.62		$200.00		$ 39.65						
June			$250.00		$ 58.11						
July			$150.00		$ 77.21						
August	$206.68		$120.00		$163.09				$3.50		$253.27

Strand: Number (Number Operations)
Students will:
- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
4. (C4–3) Solve budget problems, using graphs and tables to communicate solutions. [C, PS, T, V]	4.1 Calculate the cost of running a car for a year. Decide how to classify each cost, how to collect the data and how to display the results. 4.2 As a project, prepare a budget for one of the following: a) the family b) a working single person/yourself c) a vacation d) a fishing/hunting/shopping trip. 4.3 Analyze a budget for a school or municipality. 4.4 The diagram shows Julie's monthly budget of $1200. She wants to move to an apartment that costs $450 per month. Construct a new budget that will include her rent. Explain the changes that Julie could make.

Julie Barnes' Monthly Budget

Total = $1200

General and Specific Outcomes	Illustrative Examples [Discretionary]
5. (C4–4) Plot and describe data of exponential form, using appropriate scales. [C, T, V]	5.1 Using the information below, create a graph; plot the number of bacteria on the vertical axis and time on the horizontal axis. Use the graph to predict the bacteria population by 12:00 p.m.

Time	Number of bacteria
8:00 a.m.	5
8:30 a.m.	10
9:00 a.m.	20
9:30 a.m.	40
10:00 a.m.	80
10:30 a.m.	160

Strand: Number (Number Operations)
Students will:
- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
	5.2 The growth of a $500 RRSP invested at an interest rate of 9% per year, compounded annually, is shown on the right. a) Plot this data. b) Estimate the time needed for the RRSP to reach $1000. c) Determine the value of the RRSP after 10 years. d) Using a spreadsheet, estimate the value of the RRSP at the time of your retirement.
6. (C4–5) Solve investment and credit problems involving simple and compound interest. [CN, PS, T]	6.1 Calculate the amount, after various time periods; e.g., 1 year, 5 years, 10 years, of a deposit of $1000. Assume the current nominal annual interest when the interest is compounded: a) annually b) monthly c) daily. Discuss the effects of compound interest.
	6.2 a) Investigate the current GIC annual interest rate. If $2000 were deposited, for ten years, how much interest would be earned? b) A second bank offers an interest rate of 0.5% less per year, compounded quarterly. If $2000 were deposited, for ten years, how much interest would be earned? c) Discuss the results.
	6.3 Calculate the interest paid on various forms of credit. Items for discussion may include such things as: rate of interest, payment options, methods of calculating interest. a) credit cards b) loans c) mortgages
	6.4 You buy a bond for $1000.00 at 5.25% interest for the first year. The interest rate increases by 1% each year. Calculate the total interest you would receive after 5 years, assuming the interest earned is paid to you annually and is not rolled in to the next year's investment.

Time (years)	Value ($)
0	500
1	545
2	594
3	648
4	706
5	769

Strand: Number (Number Operations)
Students will:
- demonstrate an understanding of and proficiency with calculations
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[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
	6.5

	Card 1	Card 2
Balance Owing	$750.00	$750.00
Interest Rate Per Year	15%	18%
Annual Card Fee	$36.00	$0.00

Use the information from the chart above to answer the following questions.

a) Find the interest charged on both credit cards for the month, using the formula:

Interest = Principal × Interest Rate × Time

Note: Principal = Balance Owing

b) Including the card fee, which card is cheaper to use to make a cash advance of $750.00 that is paid back after one month?

Strand: Number (Number Operations)
Students will:
- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes

General Outcome

Modify an existing spreadsheet to make and justify financial decisions.

Specific Outcomes

7. (A8–1) Modify a spreadsheet template to allow users to input their own variables. [C, PS, T]

Illustrative Examples [Discretionary]

7.1 Modify the following table to show:
   a) change in unit price
   b) change in quantity.

ACME AUTO PARTS

Customer Inquiries

Item No.	Auto Parts	Quantity	Unit Price	Total	Labour	
1	Brake Pads	1	26.34	26.34	O/H Front Brakes	
2	Wheel Seals	2	5.25	10.50	1.5 hrs @ $57.00/hr	85.50
3	Rotor	1	30.16	30.16	Machined and	
					Replaced Rotor	10.00
					Total Labour	95.50
			Total Parts	67.00	Total Parts	67.00
					Subtotal	162.50
					GST (7%)	11.38
					TOTAL	$173.88

7.2 Roger kept good records of the costs of running his compact car for the year. He found the following costs.

Cost Category	Cost Incurred
Depreciation	$2650.00
Insurance and registration	$840.00
Tires	4 at $72.50 each
Oil changes	5 at $32.85 each
Gasoline	3340 litres at 51.9¢ per litre
Repairs	$115.00 for a tune-up $277.50 for brakes
Parking	$44.50 per month for 12 months
Washing	27 washes at $4.25 each
Interest on borrowed money	$1145.00

Enter this data into a spreadsheet template.
   a) What was the total cost of running Roger's car for the year?
   b) If the price of gasoline increased to 58.6¢ per litre, how much **extra** would it cost Roger to run his car for the year?
   c) Display the cost record as a graph.

Strand: Number (Number Operations)
Students will:
- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
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		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
8. (A8–3) Use spreadsheet software applications to analyze leasing or buying a decreasing asset (vehicle, computer) under different sets of circumstances. [C, PS, T]	8.1 A car costs \$15 366.05. A lease for the car runs for 36 months at \$305 per month, with a down payment of \$1105, a lease-end value of \$7105 and an interest rate of 11.6%. The mileage restriction is 25 000 km/year. Excess mileage will cost 8¢/km. Maintenance is the purchaser's responsibility. Set up a spreadsheet to include the monthly value of the opening balance, payment, interest rate, interest paid, principal paid and closing balance. a) If the vehicle is purchased at lease-end, find the total cost. b) How much interest is paid? c) If the car has 85 827 km at the end of the lease and the car is not purchased, what will the extra kilometres cost? 8.2 Create another spreadsheet on the same car as described in illustrative example 8.1 but with a 20% down payment and an interest rate of 11.6%. Adjust the payments so that the straight purchase loan is paid totally in 36 months. Answer 8.1a and 8.1b again.
9. (A8–5) Analyze car, property or house insurance needs and premiums, using such concepts as loss, compulsory coverage, optional coverage, deductible and claims record. [CN, E, R, T]	9.1 Obtain from an insurance company the dollar values that are paid out for inexperienced drivers and for experienced drivers in the categories of liability, collision and comprehensive theft/glass. Calculate the insurance premium for \$1 000 000 liability, \$500 deductible collision and \$100 deductible comprehensive theft/glass coverage. Do the calculations twice, once for each type of driver. What change in premium would you expect, if the deductible for collision were raised to \$1000? 9.2 At what point is it worth it to drop collision coverage on an older vehicle? Show a strategy, and explain the supporting calculations.

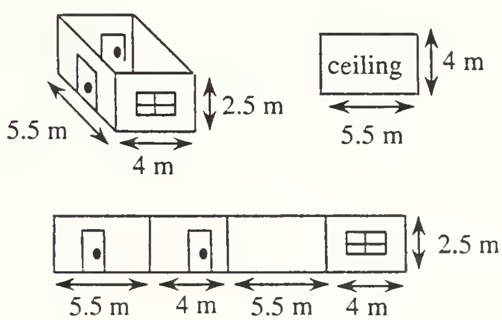
Strand: Number (Number Operations)
Students will:
- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
	9.3 How long does a security system need to be installed before the cost of the system is paid for by the savings in insurance premiums? Obtain data for your area from an insurance agent. Show a strategy, and explain the supporting calculations. 9.4 Obtain rates for property insurance for personal belongings. Estimate the value of your personal property, and calculate how much you would have to pay to insure it. Why is it important to have such insurance?

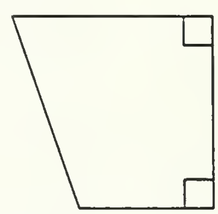
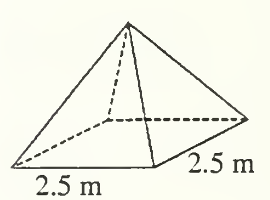
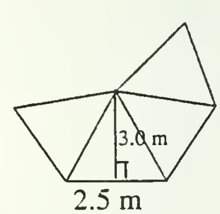
Strand: Shape and Space (Measurement)
Students will:
- describe and compare everyday phenomena, using either direct or indirect measurement.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Use measuring devices to make estimates and to perform calculations in solving problems. **Specific Outcomes** 1. (A1–3) Solve problems involving length, area, volume, time, mass and rates derived from these. [C, E, PS]	1.1 Cut some cereal boxes to form nets. a) How many faces does each have? b) What shape are the faces? c) Are any of the faces the same size? d) How could you find the area of each face? Use the data you collect to make a rule for finding the surface area of a right prism. Use your rule to find which of the cereal boxes has the greatest surface area. 1.2 Collect some cardboard cylinders that have lids. Cut the cylinders to form nets. a) How many faces does each have? b) What shape are the faces? c) Are any of the faces identical? d) Could you find the area of each face? Use the data you collect to make a rule for finding the surface area of a cylinder. Use your rule to find the surface areas of the cylinders. 1.3 Amy and John are planning to paint their apartment. The dimensions of the apartment are as follows.  (continued)

Strand: Shape and Space (Measurement)
Students will:
• describe and compare everyday phenomena, using either direct or indirect measurement.

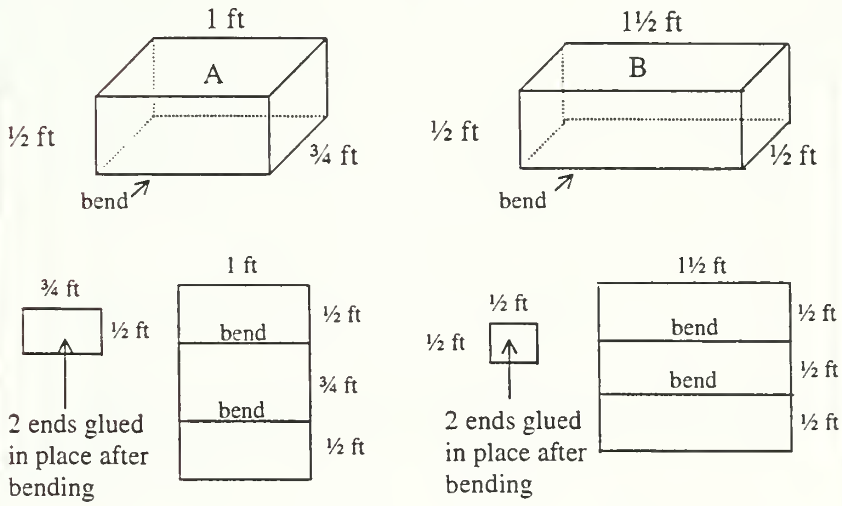
[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
	1.3 (continued) There are two doors, each 1 m x 2 m, and one window that is 50 cm x 125 cm. a) Determine the area of the ceiling to be painted. b) Determine the total area of the walls to be painted. c) What is the total area to be painted? d) If 1L of paint covers 9.5 m^2, determine how many litres will be needed to paint the apartment. e) If a painter is able to paint 3 m^2 in 10 minutes, how long will it take to paint the room? 1.4 A person buys a property that is irregularly shaped. See scale drawing below.  1 cm : 10 m What is the total area, in m^2, of the lot? 1.5 A sheet metal worker must fabricate a pyramidal cap for a square column. The base of the cap is 2.5 m by 2.5 m and the height of each triangle is 3.0 m. Determine the area of material needed.  

Strand: Shape and Space (Measurement)
Students will:
- describe and compare everyday phenomena, using either direct or indirect measurement.

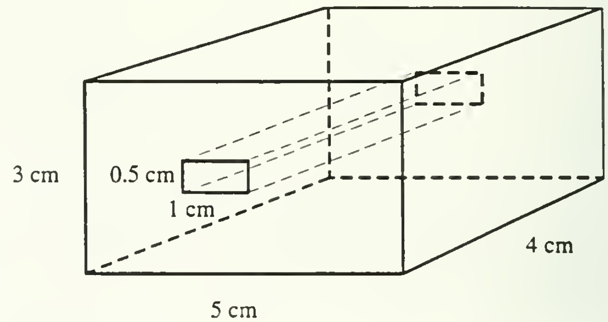
[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
	1.6 A one cubic centimetre container holds 1 mL of water, which has a mass of 1 g. a) A fish tank at the aquarium is 150 cm long and 60 cm wide. How many millilitres of water are in the tank if it is filled to a depth of 25 cm? b) How many litres is this? c) In kilograms, what is the mass of water in this tank? Why might this be important to know?
2. (A1–4) Interpret drawings, and use the information to solve problems. [C, PS, V]	2.1 Use the drawings below to answer the following questions. a) A box manufacturing company produces 2 open boxes, as shown below. Are the volumes identical? Explain your reasoning. b) If the material cost is $4.50/ft^2 and glue costs $0.70/ft, which box is cheaper to make? How much do you save? c) If a company manufactures 1000 of the cheaper boxes in a production run, how much money is saved?



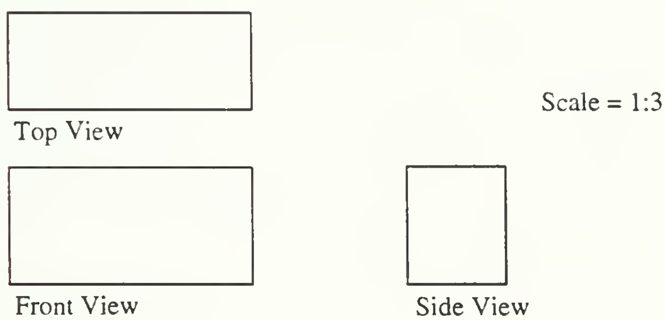
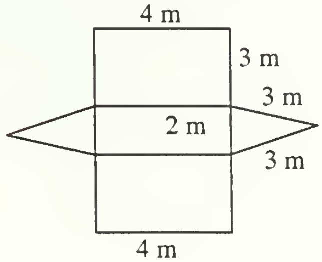
Strand: Shape and Space (Measurement)
Students will:
• describe and compare everyday phenomena, using either direct or indirect measurement.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
	2.2 First estimate, and then find, the volume and the surface area of the figure below. The figure is a 3 cm × 4 cm × 5 cm solid block of wood with a 1 cm × 0.5 cm × 4 cm hole cut in it. 

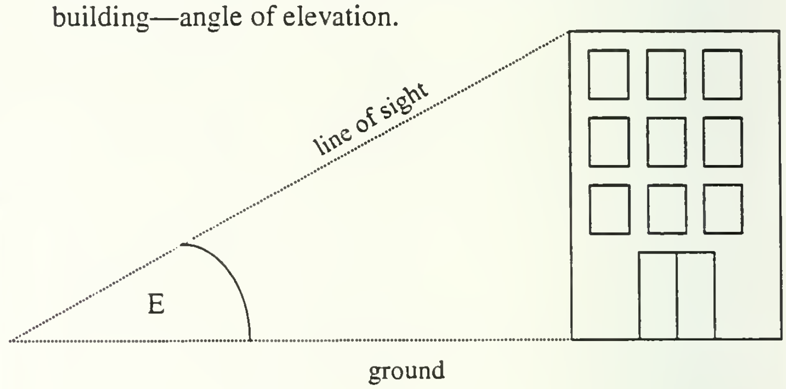
Strand: Shape and Space (Measurement)
Students will:
- describe and compare everyday phenomena, using either direct or indirect measurement.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Demonstrate an understanding of scale factors, and their interrelationship with the dimensions of similar shapes and objects. **Specific Outcomes**	
3. (A3–1) Enlarge or reduce a dimensioned object, according to a specified scale. [C, CN, PS, V]	3.1 A classroom has dimensions of 30 feet by 25 feet. Produce a scale drawing of the classroom to a scale of 5 feet:1 inch. 3.2 Using surveyor's chains, tapes or other linear measuring devices, measure a chosen plot of land, and calculate its area. Make a scale drawing, using the same measurement system for the drawing as was used with the measurement instruments. 3.3 From the scale drawing below, construct an actual sized model of the box.  3.4 To better visualize an object, designers often build models. Build a model of the tent that is shown in the net below, using the scale 10 cm = 1 m. 

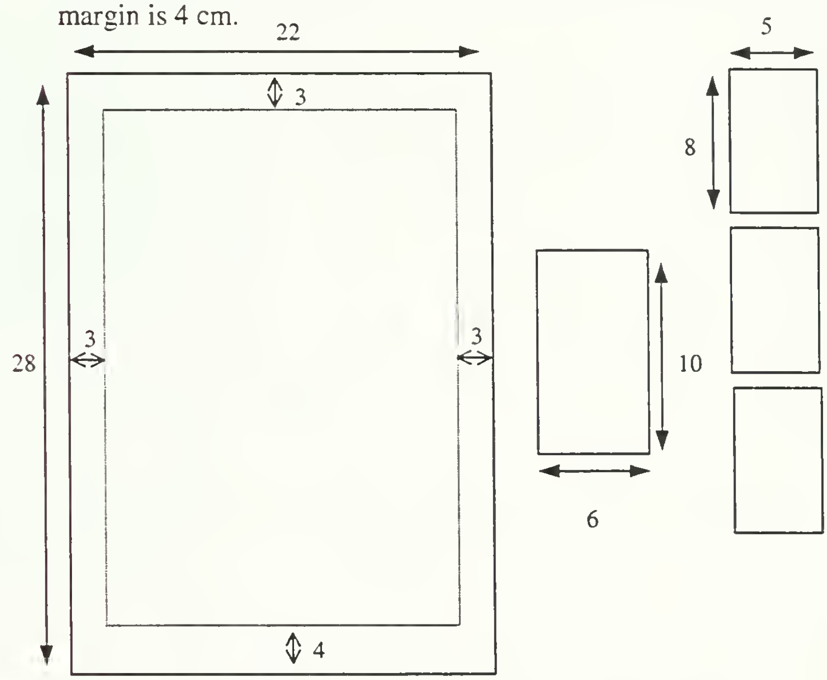
Strand: Shape and Space (Measurement)
Students will:
- describe and compare everyday phenomena, using either direct or indirect measurement.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Use measuring devices to make estimates and to perform calculations in solving problems. **Specific Outcomes** 4. (A3–4) Design an appropriate measuring process or device to solve a problem. [E, PS, V]	4.1 Use right angles to measure base placement on a softball or baseball diamond. 4.2 Design a device to paint the circles on the ice at a curling rink. 4.3 Design an instrument you can use to determine the angle formed between the ground and the line of sight to the top of a tall building—angle of elevation.  E = angle of elevation 4.4 To calculate the number of students in your school who smoke, survey a class and use this data to estimate the number of smokers in your school. 4.5 You have just bought an odometer to go with your bike. In order for the odometer to work properly, you need to enter the circumference of the tire to determine how far it will travel in one revolution. If you bought a 26 inch bike tire, what number do you input into your bike's odometer?

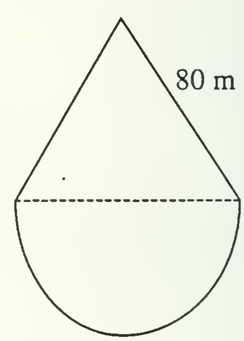
Strand: Shape and Space (3-D Objects and 2-D Shapes)
Students will:
- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

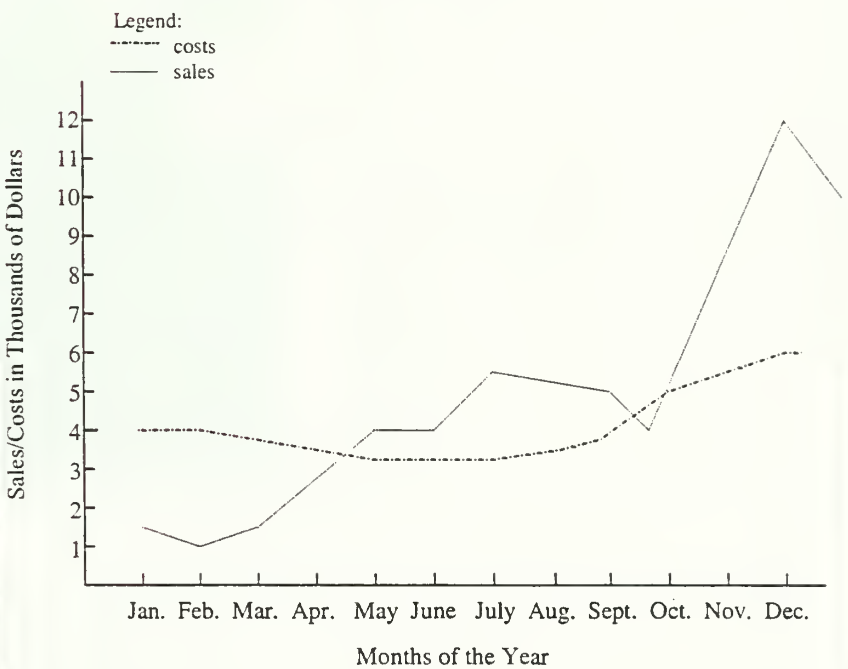
General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Develop and apply the geometric properties of circles and polygons to solve problems. **Specific Outcomes** 5. (A3–5) Use properties of circles and polygons to solve design and layout problems. [CN, PS, V]	5.1 You are preparing a school newsletter for parents. A standard sheet of paper is 28 cm by 22 cm, as is shown in the diagram below. You need to paste one table that is 10 cm by 6 cm and three tables that are 8 cm by 5 cm each onto the paper. The left, right and top margins on the page are 3 cm, and the bottom margin is 4 cm.  a) Prepare two possible layouts, assuming that the tables must be pasted with their long sides parallel to the left and right margins. b) Prepare two additional layouts with the long sides of the tables parallel to the top edge of the paper. c) What area is left over for including text on the page? Is this a reasonable amount? What percentage of the total area of the page (within the margins) is this?

Strand: Shape and Space (3-D Objects and 2-D Shapes)
Students will:
- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
	5.2 A school has 293 students, all of whom have pictures to be put in the yearbook. The yearbook pages are 9½ inches by 12 inches. The margins around the edge of the page must be 1 inch. Each photograph is 2 inches high and 1½ inches wide. The minimum space between the edges of the pictures is ½ an inch. a) What is the maximum number of photographs that can be put on a single page? b) Design a layout where: • the number of pages is divisible by 8 • the least number of pages is used • each page, except the last, has the same number of photographs. 5.3 A cylindrical can is 12 cm high and 6 cm in diameter. The can is closed, top and bottom. It is cut from a rectangular sheet of metal, and then the pieces are sealed together to form the can. a) Determine the dimensions of a rectangle that can be used to make one can. b) What area of the metal is wasted? c) What percentage of the metal is wasted? d) Share the results with your classmates. What are the dimensions of the rectangle that can be used to make one can with the least amount of waste? 5.4 The design of a water tank for seals is shown on the right. It is in the shape of an equilateral triangle with a semicircle attached to one side. Find the length of a guardrail around the whole tank. 

Strand: Statistics and Probability (Data Analysis) *Students will:* • collect, display and analyze data to make predictions about a population.	[C] Communication [CN] Connections [E] Estimation and Mental Mathematics [PS] Problem Solving [R] Reasoning [T] Technology [V] Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Analyze graphs or charts of given situations to derive specific information. **Specific Outcomes** 1. (A4–1) Extract information from given graphs of discrete or continuous data, using: • time series • continuous data • contour lines. [C, CN, E, PS, R, V]	1.1 PROFIT/LOSS CYCLE FOR A DEPARTMENT STORE  A department store may experience "peaks" and "troughs" in its revenue (sales). Christmas season and summer holidays are the two strongest periods. January to April can be the weakest period. If net profits are greater than net losses over the year, the business can stay in operation. a) During periods of net loss, what might the business do for finances? b) Over which of the two curves, sales or costs, does the business have the most managerial control? c) Discuss the net profit for May.

Strand: Statistics and Probability (Data Analysis)
Students will:
- collect, display and analyze data to make predictions about a population.

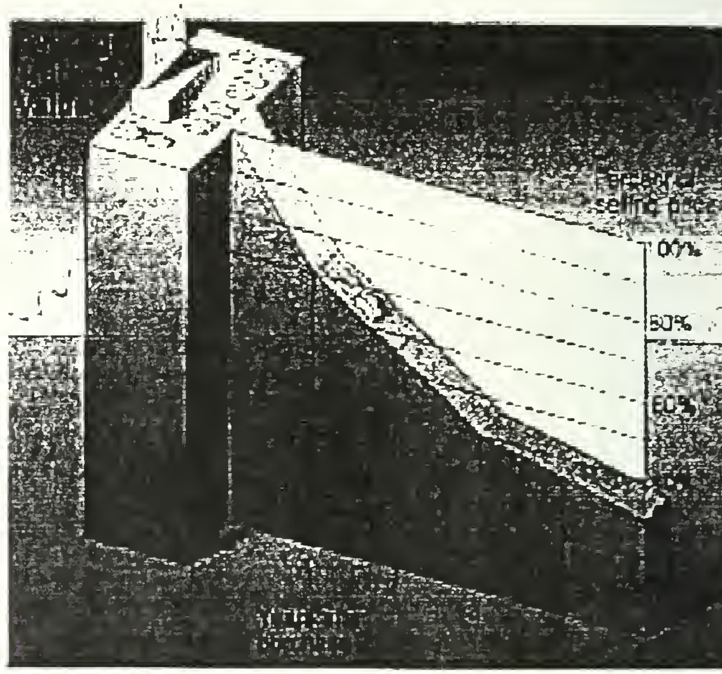
[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General and Specific Outcomes

2. (A4–2) Draw and validate inferences, including interpolations and extrapolations, from graphical and tabular data.
[CN, E, PS, V]

Illustrative Examples [Discretionary]

2.1 The trade-in value of a car decreases yearly. This loss in value is called depreciation and is illustrated in the graph below. The per cents given in the graph are based on the price of the car when it was new.



Carli et al., *Consumer and Career Mathematics*, p. 274. Reprinted and adapted with permission.

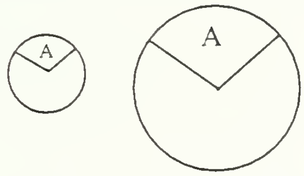
a) Mark and Cathy bought a new car for $18 900 two years ago. What is the approximate trade-in value of the car, and how much has the car depreciated?
b) Find the value of the car after 7 years.
c) Make a generalization about the depreciation of a new car over several years.

Strand: Statistics and Probability (Data Analysis)
Students will:
- collect, display and analyze data to make predictions about a population.

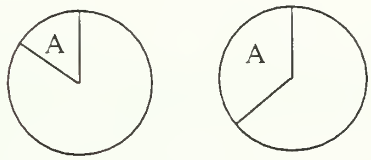
[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
3. (A4–3) Design different ways of presenting data and analyzing results, by focusing on the truthful display of data and the clarity of presentation. [C, CN, T, V]	3.1 The amount of money Company A made doubled in 6 months. The following 4 graphs were given.

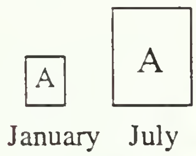
i)



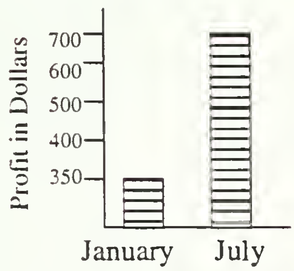
ii)



iii)



iv)



a) Which graph best represents the situation?
b) Explain what is wrong with each of the other graphical representations and why they do not represent the situation.

3.2 The value of the Canadian dollar in terms of US cents is given in the following chart. These values are recorded in graphs A and B, which use different vertical scales.
a) Do both graphs display the same data?
b) Which graph is easier to read?
c) Compare the vertical scales. How do they differ?
d) What message is delivered by Graph A? by Graph B? Discuss the differences.

(continued)

Strand: Statistics and Probability (Data Analysis)
Students will:
- collect, display and analyze data to make predictions about a population.

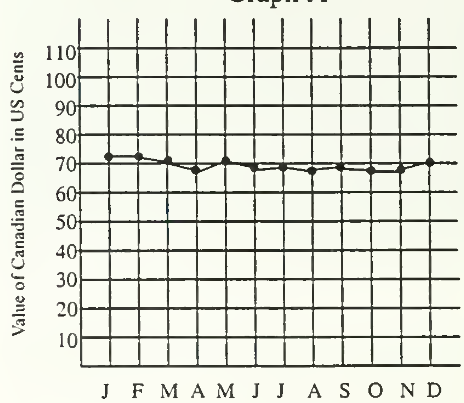
[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
	3.2 (continued)

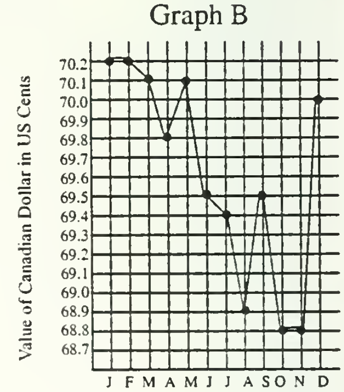
Value of Canadian Dollar in US Cents

January	70.2
February	70.2
March	70.1
April	69.8
May	70.1
June	69.5
July	69.4
August	68.9
September	69.5
October	68.8
November	68.8
December	70.0

Graph A



Graph B



Strand: Statistics and Probability (Chance and Uncertainty)
Students will:
- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Make and analyze decisions, using expected gains and losses, based on the probabilities of simple events. **Specific Outcomes**	
4. (P2–6) Connect probabilities to calculated expected gains or losses. [CN, PS, R, V]	4.1 You are designing a coin-tossing game to be used at the school fund-raiser. Students pay 25 cents to toss three coins at the same time—a quarter, a dime and a nickel. They can keep those coins that turn up heads. a) Toss the three coins. In the second column of the chart below, record the coins with heads. b) Calculate the payout for the toss, and record the payout in the third column of the chart. c) Calculate the amount raised or lost for the toss, and record this in the fourth column. d) Repeat steps a) to c) until the chart is complete. The first two lines of the chart have been filled in already.

Results Chart

Toss Number	Coins with Heads	Payout on this Toss	Amount Raised or Lost on this Toss
1	quarter and dime	35 cents	10 cents lost
2	dime and nickel	15 cents	10 cents raised
3			
4			
5			
6			
7			
8			
9			
10			

e) Use your results from the chart to estimate the expected amount the school would raise if the game were played with 200 tosses.
f) Use your knowledge of probability to calculate—not predict—the average payout per toss for this game.

Strand: Statistics and Probability (Chance and Uncertainty)
Students will:
- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
	4.2 Sherry takes a 100-item multiple-choice examination. Each item has four possible choices. She knows 68 of the answers and guesses randomly at the other 32. Calculate her expected number of correct answers.

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